Default Risk Heterogeneity and Borrower Selection in the Mortgage Market*

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Abstract

This paper studies limits on debt payment-to-income ratios that vary in how strictly they are applied in the mortgage market. I document that, under such flexibility, a higher debt payment-to-income ratio need not imply greater default risk. To explain this, I propose an incomplete-markets life-cycle model in which competitive lenders have discretion to relax a debt payment-to-income constraint for sufficiently creditworthy households in equilibrium. The model accounts for observed patterns of default risk heterogeneity and highlights welfare gains from regulations that use a loan’s price to proxy ability to repay instead of the debt payment-to-income ratio.

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1 Introduction

In the aftermath of the Great Recession, policymakers have introduced new macroprudential measures designed to reduce mortgage foreclosure and manage housing price risk. In the United States, a crucial part of these reforms was the imposition of a limit on the debt payment-to-income (DTI) ratio of borrowers. I document that, in practice, how strictly this limit is applied across households is non-uniform. However, in almost all structural macroeconomic models used to study default, borrowing constraints are tight, making it difficult to conduct policy counterfactuals. The main goal of this paper is to study the efficacy and welfare effects of DTI limits in the presence of realistic household heterogeneity. I exploit variation from an institutional change in underwriting standards allowing lenders to originate mortgages that exceed a statutory DTI limit in order to establish that a higher DTI ratio does not necessarily imply greater probability of default. To explain this, I incorporate a tractable screening technology and loan contract choice in an otherwise standard incomplete-markets life-cycle model with a competitive mortgage market. The calibrated model rationalizes empirical features of default risk heterogeneity and borrower selection in the U.S. mortgage market that elude existing quantitative frameworks in the literature. Finally, the model highlights scope for welfare improvements that arise from regulating mortgage credit on the basis of a loan’s price rather than the household’s DTI ratio.

Using data on conforming mortgage originations, I find that default risk exhibits a sharp discontinuity at the current DTI limit: compared to borrowers at the limit, those above it have higher credit scores and receive lower interest rates on their loans. Thus, conditional on having large observable DTI ratios, borrowers still exhibit substantial heterogeneity in their ability to repay. I also show that a fraction of borrowers on the DTI limit are no more likely to default than borrowers above it. These observations reflect the additional cost of underwriting mortgages to riskier borrowers whose DTI ratios exceed the statutory limit. As a result, more creditworthy households select into loans with larger DTI ratios.

To rationalize these findings and conduct policy evaluation, I study an incomplete-markets life-cycle model with competitive loan pricing and long-term illiquid mortgages à la Kaplan, Mitman and Violante (2020) that is augmented with two new features. The first is a screening technology that tractably models the discretion of lenders to exempt households with sufficiently low probability of default from a DTI limit. This is motivated by the observation that some borrowers in the data appear constrained by the DTI limit but others do not. The second feature is a discrete contract type that represents loan-specific costs incurred by lenders if they originate a “risky” mortgage. This allows the model to capture the relevant trade-offs embedded in reforms to DTI limits proposed since the Great

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Recession, making it an appropriate setting for policy counterfactuals.

The calibrated model successfully matches observed heterogeneity in default risk conditional on borrowers’ DTI ratios. The loan screening technology generates borrower selection effects crucial for this result. In the standard version of the model without this feature, the equilibrium default risk of borrowers is monotonically increasing in their DTI ratio, and the variance of default risk across borrowers is counterfactually large. The default risk threshold generates a distribution in which only households with relatively low endogenous default risk hold mortgages with DTI ratios above the limit. Of the households screened out of DTI-unconstrained loans, those with the highest default probabilities exit the mortgage market altogether, lowering the dispersion in default risk across the population of borrowers in the stationary distribution. The remainder reduce their desired loan size until their DTI ratios fall just below the statutory maximum. Thus, the model replicates the discontinuity in default risk at the DTI limit documented in the data.

By uncovering the counterfactual loan choices of households for whom the DTI limit binds, the structural model explains why households with large desired DTI ratios vary in their propensity to default. Households constrained by the limit along the extensive margin of debt are primarily existing renters who, in the absence of this constraint, would have financed a new house purchase with a large loan. Their high default risk reflects their low home equity conditional on becoming owners. They are vulnerable to adverse shocks to cash on hand that could make future default optimal. Households constrained along the intensive margin are mostly liquidity-constrained owners who refinance their loans for precautionary reasons. Their lower default risk reflects their substantial home equity, which gives them the option to liquidate their houses rather than default in response to future negative shocks.

A structural model consistent with the observed selection of borrowers into loans with large DTI ratios is vital for evaluating the welfare implications of reforms to mortgage underwriting standards. To demonstrate this, I use the calibrated model to quantify the aggregate and distributional effects of two proposed versions of the Dodd-Frank Act’s ability-to-repay rule that would overlay the current DTI limit. Both proposals assign greater legal liability to lenders if a borrower defaults on a mortgage originated with a risky feature but differ in what that feature is. The original rule penalized mortgages with a DTI ratio above a set threshold, whereas the revised rule penalizes loans with a high interest rate spread. The loan contract types in the model parsimoniously capture the effects of both policies on allocations and welfare. Relative to the original rule, the revised rule improves aggregate welfare because the pass-through of the legal penalty to households’ cost of borrowing in the model is weaker under it. Intuitively, a household who demands a larger loan mechanically raises their DTI ratio at origination, potentially incurring the legal cost prescribed by the
original rule, even if their expected probability of default is comparatively low. By contrast, unless the household’s default risk is already high, the marginal increase in loan size has little effect on their equilibrium interest rate, making it less likely their loan violates the revised rule’s price-based threshold. I show that welfare gains are especially large for households who rent under the original rule because their large implied DTI ratio makes obtaining a new loan too costly but can feasibly own under the revised rule.

The paper proceeds as follows. In Section 2, I connect my paper to the relevant literature. In Section 3, I present evidence on the heterogeneity of default risk with respect to DTI ratios. In Section 4, I develop a theoretical model that explains these facts. In Section 5, I discuss calibration and validate model fit. In Section 6, I show that the calibrated model accounts for empirical patterns of loan selection and borrower heterogeneity. In Section 7, I use the model to study reforms to underwriting standards and find that a policy regulating mortgage credit on the basis of price, instead of DTI ratios, improves welfare. Section 8 concludes.

2 Related literature

On the theoretical side, my paper builds on a growing body of macroeconomic research that features uninsurable idiosyncratic risk, illiquid housing wealth, long-term debt, and borrowing constraints in structural models of housing and mortgage markets. My theoretical environment has much in common with models of household consumption that distinguish between liquid and illiquid assets. With its inclusion of competitive loan pricing and endogenous default, my paper speaks to a related literature that uses equilibrium models to study bankruptcy and foreclosure.

My empirical contribution is to document that an influential limit on debt payment-to-income ratios in the U.S. mortgage market is not applied uniformly across households and that relatively more creditworthy borrowers hold loans that violate this limit. Thus, a high DTI ratio at origination does not necessarily imply an inability to repay. The benchmark structural model used in macroeconomics to study the mortgage market—in which a household’s choice of debt must always satisfy one or more ad hoc constraints at origination—is by construction inconsistent with this. Consequently, my theoretical contribution is to embed a simple loan screening technology in the standard framework

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1 Davis and Van Nieuwerburgh (2014) and Piazzesi and Schneider (2016) provide overviews of this literature. For specific examples, see Beraja et al. (2019), Berger et al. (2017), Boar, Gorea and Midrigan (2020), Guren, Krishnamurthy and McQuade (2021), Guren et al. (2021), Kaplan, Mitman and Violante (2020), and Wong (2021).


whereby the strictness of a DTI constraint at origination is a function of a borrower’s endogenous probability of default. This gives rise to a stationary distribution in which the constraint binds for some but not all agents with high desired DTI ratios in equilibrium.4

My paper adds to ongoing work that studies the effects of \textit{ex ante} regulations in the U.S. mortgage market implemented or proposed since the Great Recession.5 Because DTI ratios and default probabilities are equilibrium objects, my approach uses a calibrated structural model to infer unobservable household characteristics that rationalize documented patterns of loan choice and to conduct policy counterfactuals. My quantitative results reveal the rich heterogeneity that exists across households with similar desired DTI ratios and the importance of accounting for it when evaluating household leverage regulations.

3 Empirical evidence on DTI ratios and default risk

The conforming mortgage market has operated under a limit of 45% on debt payment-to-income ratios since 2009.6 Despite this, a significant share of originations violate this requirement. Where previous work has focused on the bunching of loans at the DTI limit as evidence of its influence on household leverage, I exploit differences between mortgages on either side of the limit to establish that there is significant heterogeneity in how binding this constraint is across households. Before 2009, default risk is strictly increasing in the DTI ratio at origination. Afterwards, a discontinuity emerges at the 45% cutoff: relative to borrowers at the limit, those above have significantly higher credit scores and face lower costs of borrowing in equilibrium. Thus, among households most directly affected by the DTI limit, a higher DTI ratio need not imply a lower ability to repay. I also show that the distributions of credit scores associated with loans on either side of the limit overlap considerably. Some borrowers for whom the DTI limit binds are no more likely to default than those exempt from it, suggesting that some relatively creditworthy households are constrained by it.

3.1 Institutional background on DTI limits and data description

In March 2009, Freddie Mac revised its underwriting standards to include a 45% maximum back-end debt payment-to-income ratio at origination for manually underwritten

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4Corbae and Quintin (2015) propose a theory in which borrowers choose between low- or high-down payment fixed-rate mortgages. Their model place strong restrictions on housing tenure and loan adjustment choices that I relax. Chambers, Garriga and Schlagenhauf (2009) study loan structure in general equilibrium but abstract from default. Gete and Zecchetto (2018) study an environment with different loan types; however, they model mortgages as one-period liquid debt and abstract from a life-cycle savings motive.


6A conforming loan is a mortgage satisfying the purchase requirements of Fannie Mae and Freddie Mac.
}
mortgages.\footnote{This policy was announced on November 24, 2008. See https://guide.freddiemac.com/app/guide/section/5401.2 for a text of the policy. The back-end DTI ratio includes in the numerator payments on all outstanding liabilities of the borrower and housing expenses. Manual underwriting is the process by which a lender assesses the loan application in accordance with GSE underwriting requirements.} Freddie Mac is one of two large government-sponsored enterprises (GSEs) that purchase mortgage originations from primary lenders and issue securities backed by those mortgages to investors on the secondary market. Lenders have some discretion to relax the DTI limit because there are extenuating circumstances under which Freddie Mac will purchase a mortgage originated with a DTI ratio above 45%. These may include a borrower possessing enough liquid assets to constitute an ability to repay regardless of income; a down payment on the purchase of a property of at least 25%; or a strong credit score, defined as 740 or higher, combined with the lender’s written assurance that “the borrower’s credit reputation is excellent.” Exercising this discretion is not costless to the lender. Since the Great Recession, lenders have had strong incentives to adhere to GSE purchase requirements. The GSEs have more aggressively enforced representations and warranties and reserve the right to force lenders to buy back mortgages—particularly those in delinquency or default—that were improperly underwritten. In response, lenders have applied stricter underwriting standards to riskier loans and incurred higher origination costs.\footnote{Over this time period, the GSEs have purchased or guaranteed around two-thirds of mortgage originations. Private-label securitization concurrently accounted for less than 1% of originations volume. See https://www.urban.org/sites/default/files/publication/102776/august-chartbook-2020.pdf. Fuster, Lo and Willen (2017) document rising costs of financial intermediation in the mortgage market between 2008–2013. Goodman and Zhu (2013) discuss putback risk.}

For the empirical analysis, I use the Freddie Mac Single Family Loan-Level Dataset. It contains quarterly loan-level data on fully amortized fixed-rate mortgage originations with full documentation that have been purchased or guaranteed by Freddie Mac since 1999. I limit my analysis to mortgages originated between 2005 and 2016 with 30-year terms that are collateralized by owner-occupied housing and have non-missing DTI ratios, credit scores, interest rates, and loan-to-value (LTV) ratios. This results in a sample of approximately 10 million loans.\footnote{I provide descriptive statistics in Appendix A.1. If I do not restrict this sample to 30-year mortgages, average maturity is 26.3 years and loans with 30-year terms account for 74% of them. I show in Appendix A.3 that my empirical findings are robust to the inclusion of mortgages with varying maturity.} I will refer to loans that do and do not meet a statutory DTI limit as low-DTI loans and high-DTI loans, respectively, for the rest of the paper.

### 3.2 Selection of creditworthy borrowers into high-DTI loans

Although the share of mortgages with a debt payment-to-income ratio above 45% has declined relative to its pre-policy level, Figure 1 shows that the high-DTI category has
accounted for around 9% of conforming loans since the DTI limit was introduced. This indicates that lenders indeed exercise the discretion permitted by underwriting guidelines to relax the DTI requirement and suggests there is important variation in how strictly this requirement is applied. This is corroborated by a marked change in the distribution of borrower default risk across DTI ratios. In the pre-policy period, high-DTI borrowers had a mean credit score at origination of 716, compared to 726 for low-DTI borrowers. This observation underlies common justifications for using DTI limits as a policy instrument for reducing household leverage and mortgage default. While the average credit score of borrowers obtaining new loans has risen substantially since the Great Recession, high-DTI borrowers have become even more creditworthy relative to their pre-policy level than low-DTI borrowers. Mean credit scores on either side of the limit are nearly identical in the post-policy period, suggesting that DTI ratios have become less correlated with default risk.

Figure 2 confirms this intuition by showing that, under current regulations, borrower default risk is non-monotonic in DTI ratios in the neighborhood of the statutory limit. In the left-hand panel, I plot credit scores, residualized with respect to an aggregate time trend, as

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\[\text{In Appendix A.1, I plot distributions of DTI ratios at origination by year and provide evidence that the 45% DTI requirement effectively applies to all GSE-eligible loans, not merely those bought by Freddie Mac.}\]

\[\text{For example, the Consumer Financial Protection Bureau (2013) writes, “...the Bureau believes that DTI is an indicator of the consumer’s ability to repay. All things being equal, consumers carrying loans with higher DTI ratios will be less able to absorb any such shocks and are more likely to default.”}\]
Figure 2. Default risk as a function of the DTI ratio at origination

Notes: The credit score and interest rate are each residualized with respect to a vector of quarter dummy variables. Averages are computed for each 1-percentage point DTI bin for pre- and post-2009Q1 observations separately. The vertical dashed black line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.

A function of DTI ratios for loans originated before and after the implementation of the DTI limit in 2009. Before 2009, a borrower’s credit score was monotonically decreasing in their DTI ratio, suggesting that the DTI ratio was a good observable proxy of default risk. From 2009 onward, however, a clear discontinuity in credit scores emerges at the statutory DTI limit as borrowers above the limit become more creditworthy while borrowers immediately below it become less creditworthy. One way to contextualize the relative improvement in the ability to repay of high-DTI borrowers is to note that borrowers with a DTI ratio of 50% have credit scores comparable to those of borrowers with a DTI ratio of 30%. To the left of the limit, though, credit scores remain strictly decreasing in the DTI ratio, and borrowers with DTI ratios exactly equal to 45% have the lowest credit scores of all. The observed non-monotonicity of default risk with respect to DTI ratios in the data indicates that, in equilibrium, lenders only exercise the discretion they have to originate high-DTI loans in favor of borrowers with sufficient creditworthiness.

The right-hand panel of Figure 2 shows that variation in ex ante default risk is reflected in the interest rate that borrowers receive on their loans. Before 2009, it is increasing in DTI ratios through the 45% cutoff. After 2009, it becomes discontinuous at this point: borrowers to the right of the DTI limit receive below-average interest rates, while those immediately to the left receive above-average interest rates. This finding is especially striking because
3.3 Heterogeneity in default risk among borrowers on the DTI limit

To summarize, the findings from Section 3.2 show that lenders exercise the discretion they have to originate high-DTI loans only for borrowers whose ability to repay they judge to be beyond doubt. The DTI requirement, however, still applies to the loan choice of borrowers who fall short of this implicit creditworthiness standard. As a result, the borrowers for whom the limit appears most binding—i.e., those with DTI ratios just at or below 45%—exhibit the highest \textit{ex ante} default risk on average.

Next, I document that, despite the difference in mean default risk on either side of the DTI limit, borrowers at the limit are not uniform in their ability to repay. In Figure 3, I plot distributions of credit scores associated with mortgages originated in 2009Q1 and later into 26 credit score bins. The dashed black vertical line marks the credit score of 740. Source: Freddie Mac Single Family Loan-Level Dataset.

Notes: The distributions are constructed by sorting mortgages originated in 2009Q1 and later into 26 credit score bins. The dashed black vertical line marks the credit score of 740. Source: Freddie Mac Single Family Loan-Level Dataset.

A large DTI ratio can mechanically result from a high interest rate, all else equal.\textsuperscript{12} The fact that high-DTI loans are associated with relatively lower interest rates underscores the borrower selection effects at work.\textsuperscript{13}

\textsuperscript{12}A large DTI ratio could also reflect a large initial loan size. In Appendix A.1, I verify that the DTI ratios of borrowers to the right of the 45% limit are not driven by this fact. Indeed, high-DTI loans have been associated with lower LTV ratios, relative to loans just below the limit, since 2009.

\textsuperscript{13}In Appendix A.2, I estimate a difference-in-differences regression and find that these correlations remain statistically after controlling for other observable loan-level characteristics.
separately for borrowers with DTI ratios above 45% and those with DTI ratios between 43% and 45%. It reveals the existence of significant dispersion in the default risk across borrowers with DTI ratios at the statutory limit. The credit scores of high-DTI borrowers display lower variance, which is expected given that that lenders can only qualify relatively safe borrowers for high-DTI loans.

This feature of the data provides suggestive evidence that a nontrivial share of borrowers located at the DTI limit are constrained by it despite not being observationally more likely to default than borrowers above the limit. An indication of this is the share of borrowers at the limit who have credit scores of 740 or above. Since the current limit was implemented in 2009, around 20% of mortgage originations with DTI ratios between 43% and 45% have gone to borrowers with credit scores in this range. This cutoff is relevant because, as mentioned in Section 3.1, it is one condition under which a lender may waive the DTI requirement. The data make clear that a credit score in this range is neither necessary nor sufficient for obtaining a high-DTI loan. Nonetheless, it is an observable proxy for how creditworthy a borrower may need to be to qualify for one.

Altogether, these stylized facts highlight the heterogeneity in default risk that exists among borrowers with large debt payment-to-income ratios. Relatively more creditworthy borrowers have selected into high-DTI loans since the introduction of a DTI limit in the conforming loan market, whereas borrowers located at the statutory limit represent a more heterogeneous mix of default probabilities. These observations raise further questions. First, why do households who vary in their ability to repay have similar desired DTI ratios in equilibrium? Second, how binding is the current DTI limit, and how much discretion do lenders have to relax it? Third, if large desired DTI ratios at origination are not a necessary condition for borrowers to possess a high probability of default in the post-Great Recession years, how do policies that restrict borrowing on the basis of DTI ratios affect household allocations and welfare? In the following sections, I develop a theoretical framework to rationalize the features of the data documented above and answer these questions.

4 A model of mortgage lender discretion and contract type

To explain the novel aspects of the data documented in Section 3, I introduce two features to an otherwise standard incomplete-markets life-cycle model with a competitive mortgage

\footnote{A DTI threshold of 43% is relevant in the policy counterfactual in Section 7. Restricting the comparison to borrowers who have a DTI ratio equal to 45% results produces nearly identical results.}

\footnote{To place this in perspective, each loan sold to Freddie Mac is charged a delivery fee that depends on the credit score and LTV ratio of the borrower. Conditional on the LTV ratio, delivery fees are decreasing in the credit score, with 740 representing the top tier of creditworthiness. See http://www.freddiemac.com/singlefamily/pdf/ex19.pdf.}
market. Motivated by the existence of borrowers with debt payment-to-income ratios at origination that exceed a statutory maximum, I embed in this framework a loan screening technology. This takes the form of a default risk threshold that a household must satisfy in equilibrium in order for their loan choice to not be subject to a DTI limit at origination. In the standard model, competitive financial intermediaries are risk neutral and have perfect information. As a result, the market for loans originated to households conditional on their current state clears automatically as long as a zero-expected profit condition holds. The default risk threshold models the discretion that lenders have in reality to relax the statutory DTI limit for borrowers with high enough ability to repay and tractably generates a stationary distribution in which this borrowing constraint binds for some but not all agents.

I also introduce a loan contract type in order to make the model suitable for studying policy counterfactuals. The contract type is a discrete variable that tracks whether a household’s mortgage satisfies a particular statutory requirement—e.g., regarding the DTI ratio or the interest rate—at origination. This is necessary because proposed reforms to underwriting standards aimed at reducing foreclosure operate by imposing additional loan-specific costs on lenders if a borrower defaults on a mortgage that did not meet certain conditions at origination. These reforms affect the equilibrium price and quantity of mortgage debt through its continuation value because these costs are realized in future states of the world in which a borrower optimally defaults. The contract type is general enough to nest a variety of the reforms to DTI limits considered since the Great Recession.

I develop the model in this section. I summarize its environment with emphasis on the two new features. Next, I state the optimization problems of households and lenders. I solve for the stationary recursive equilibrium, defined in Appendix B.1.16

4.1 Model overview

Time is discrete. The model features a constant population of overlapping generations of households who split their lives between working and retirement. When working, they receive age-specific endowment income subject to uninsurable idiosyncratic risk. When retired, they receive constant pension income. Households can save in both a liquid asset and illiquid housing wealth. Homeowners can borrow through illiquid long-term, fixed-rate mortgage contracts subject to borrowing limits at origination. Debtors have the option to default. Households derive utility each period from consumption of a nondurable good and housing services. They can either rent housing services or purchase a house that yields a service flow each period. House prices are subject to uninsurable idiosyncratic shocks. Homeowners also experience depreciation shocks.

16I provide a detailed description of the numerical solution algorithm in Appendix C.
The model also features a continuum of competitive, risk neutral, and infinitely lived financial intermediaries. They store the liquid savings of and supply mortgage debt to households. Because financial intermediaries observe the household’s idiosyncratic state, the equilibrium interest rate on a newly originated mortgage reflects the household’s endogenous default risk and is such that the lender makes zero expected profit on a loan-by-loan basis.

Preferences and endowments

A household maximizes expected discounted lifetime utility, defined as

$$\max_{\mathbb{E}} \left\{ \sum_{j=1}^{T} \left[ \beta^{j-1} u_j (c_j, s_j) \right] + \beta^T \nu (W_{T+1}) \right\},$$

where \( j \in \{1, 2, \ldots, T\} \) is the household’s age and \( \beta \in (0, 1) \) is the subjective discount factor. The flow utility function \( u (\cdot) \) satisfies the Inada conditions and is given by

$$u_j (c_j, s_j) = \frac{1}{1-\sigma} \left[ \alpha_j c_j^{1-\vartheta} + (1 - \alpha_j) s_j^{1-\vartheta} \right] \frac{1-\vartheta}{1-\sigma},$$

where \( c \) is nondurable consumption and \( s \) is housing services. \( \sigma > 0 \) is the coefficient of relative risk aversion, and \( 1/\vartheta > 0 \) is the elasticity of substitution between nondurable consumption and housing services. \( \alpha_j \in (0, 1) \) is the preference weight on nondurable consumption that potentially depends on age.\(^{17}\) Households have a bequest motive whereby they receive expected discounted utility from end-of-life wealth \( W_{T+1} \) according to

$$\nu (W_{T+1}) = B \mathbb{E} \left\{ \frac{W_{T+1}^{1-\sigma}}{1-\sigma} \right\}.$$

The parameter \( B > 0 \) controls the strength of the bequest motive.

A household supplies labor inelastically from age 1 until they retire at age \( T_R \). While working, a household’s log income is the sum of a deterministic component indexed by age \( \chi_j \) and an idiosyncratic component \( z \) that evolves according to the first order Markov process

$$z' = \rho z + \varepsilon', \quad \varepsilon' \text{i.i.d.} \mathcal{N} (0, \sigma^2 \varepsilon).$$

Upon retirement, a household receives a constant pension that is a deterministic function

\(^{17}\)This captures life-cycle components of demand for housing services that are not explicitly modeled.
Φ (·) of earnings at age \( T_R - 1 \). The process for log income is therefore

\[
\log y_j(z) = \begin{cases} 
\chi_j + z & \text{if } 1 \leq j \leq T_R - 1 \\
\Phi (y_{T_R-1}(z)) & \text{if } T_R \leq j \leq T.
\end{cases}
\]

**Liquid savings**

Households can save in a one-period liquid asset \( a \), subject to a no-borrowing constraint, in the form of deposits held by the financial intermediaries. Intermediaries have access to international capital markets where the net supply of safe assets determines the risk-free rate \( r > 0 \). A zero-profit condition implies households also earn return \( r \) on liquid assets.

**Housing**

Households obtain housing services through either the rental or owner-occupied housing market. If the household participates in the owner-occupied housing market, they buy a house of size \( h \) at price \( p \). They choose a house size from a discrete grid and can only own one house at a time.\(^{18}\) The supply of houses of all sizes is perfectly elastic. The house yields a one-to-one flow of services each period. Following Berger et al. (2017) and Mitman (2016), house prices are subject to an idiosyncratic shock and follow the first order Markov process

\[
\log p' = \rho \log p + \eta', \quad \eta' \text{i.i.d.} \sim \mathcal{N} (0, \sigma^2_\eta).
\]

The price affects the value of a home but not the quantity of services it generates. Households who adjust their housing stock pay a fixed transaction cost \( \kappa_h > 0 \). As in Chatterjee and Eyigungor (2015), housing stock is subject to an i.i.d. depreciation shock \( \delta \in (0, 1) \), where

\[
\delta = \begin{cases} 
\bar{\delta} & \text{with probability } \zeta \\
\tilde{\delta} & \text{with probability } 1 - \zeta
\end{cases}
\]

\(^{18}\)Piazzesi and Schneider (2016) emphasize indivisibility as a key feature of housing an asset.

\[
\text{and } \bar{\delta} > \tilde{\delta}, \text{ so that owners pay a housing maintenance cost of } \delta \phi h \text{ each period.}
\]

Otherwise, a household can rent \( s \) units of housing services at a rate \( R \phi \) per unit each period, where \( R > 0 \) denotes the exogenous rent-price ratio. The supply of rental housing is perfectly elastic. Adjusting the quantity of rental housing between periods does not incur a transaction cost, and renters do not experience depreciation shocks.
Mortgages

Owners can use their house as collateral for fixed-rate mortgage debt $m$ that is amortized over their remaining lifetime.\(^{19}\) Loan size is subject to loan-to-value and debt payment-to-income ratio limits at origination. Households borrow at an equilibrium interest rate $r_m$ that is a function of their idiosyncratic state. The characteristics of a loan at origination imply a contract type represented by $q \in \{L, H\}$ that is fixed for the duration of the loan. The current state of an age-$j$ household is therefore summarized by the vector $\omega \equiv (a, h, \delta, p, m, q, r_m, z)$.

For households whose endogenous probability of default falls below a certain threshold, the DTI limit is fully relaxed. The threshold captures the fact that, while lenders have discretion to originate high-DTI loans, doing so is prohibitively costly if a borrower is too risky.\(^{20}\) The default risk threshold tractably introduces a screening problem into the model whereby lenders only originate high-DTI loans to borrowers who, conditional on their loan choice, are sufficiently creditworthy in equilibrium.\(^{21}\)

The DTI limit’s formulation highlights the role of the screening technology. The limit,

$$\pi_{\text{min},j}(m, r_m) \leq \lambda_j(\omega) y_j(z),$$

states the minimum mortgage payment $\pi_{\text{min},j}(m, r_m)$ cannot exceed a fraction $\lambda_j(\omega)$ of the household’s income $y_j(z)$. The tightness of the DTI constraint is a function of the household’s state because it depends on their endogenous likelihood of default. I specify $\lambda_j(\omega)$ as

$$\lambda_j(\omega) = \begin{cases} 
\lambda - \varsigma_j(\omega) & \text{if } \psi_j(\omega) > \Psi \\
\infty & \text{if } \psi_j(\omega) \leq \Psi.
\end{cases} \quad (3)$$

Let $\psi_j(\omega)$ denote a household’s endogenous default risk and the parameter $\Psi \in (0, 1)$ denote the default risk threshold.\(^{22}\) If a household’s equilibrium default risk exceeds the threshold, then the DTI ratio implied by their choice of loan size must be less than $\lambda - \varsigma_j(\omega)$. $\lambda \in (0, 1)$ is the statutory DTI limit, and $\varsigma_j(\omega) \in (0, 1)$ is an exogenous offset term representing non-first mortgage liabilities and housing expenses relative to income that is a deterministic

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\(^{19}\)See Kaplan, Mitman and Violante (2020) and Wong (2021). This implies that maturity at origination is $T - j$, consistent with the observed negative correlation between age and loan duration. It ensures I do not need to track maturity as a state variable. In the calibrated model, mean loan maturity is 34 years.

\(^{20}\)Freddie Mac states, “When there is evidence of layered risk, more conservative underwriting must be undertaken to assess whether the Mortgage is acceptable for sale to Freddie Mac.” See https://guide.freddiemac.com/app/guide/section/5102.2.

\(^{21}\)The default risk threshold is isomorphic to an exogenous constraint on credit supply studied by Justiniano, Primiceri and Tambalotti (2019). In general, this literature abstracts from asymmetric information in lending markets. Exceptions include Chatterjee et al. (2020) and Guler (2015).

\(^{22}\)See Equation (11) for an expression for $\psi_j(\omega)$ after the relevant value functions have been defined.
function of the household’s state. I follow the literature in adjusting the statutory DTI limit downward in order to bring model-implied DTI ratios in line with the data because underwriting standards target a borrower’s back-end DTI ratio.\textsuperscript{23} If a household’s default risk meets the threshold, their DTI ratio is not subject to any ad hoc limit at origination.

In the baseline version of the model, the contract type is simply a function of whether the DTI ratio at origination complies with the GSEs’ statutory requirement. Thus,

$$q_j(\omega) = \begin{cases} L & \text{if } \pi_{\text{min},j}(m, r_m) \leq (\lambda - \varsigma_j(\omega)) y_j(z) \\ H & \text{if } \pi_{\text{min},j}(m, r_m) > (\lambda - \varsigma_j(\omega)) y_j(z). \end{cases}$$

However, as I will show in the counterfactual analysis in Section 7, it is straightforward to map the distinction between contracts to any mortgage characteristic at origination.

The other features of mortgage debt are common to the literature. A LTV constraint

$$m \leq \theta ph$$

limits the loan size to a fraction $\theta \in (0, 1)$ of the value of the home $ph$. A household who obtains a new loan pays a fixed transaction cost $\kappa_m > 0$ and can only hold one loan at a time. An age-$j$ borrower with predetermined debt $m$ and interest rate $r_m$ makes a loan payment of at least $\pi_{\text{min},j}(m, r_m)$ each period, where $\pi_{\text{min},j}(m, r_m)$ is computed as

$$\pi_{\text{min},j}(m, r_m) = \frac{(1 + r_m)^{T-(j-1)}}{(1 + r_m)^{T-(j-1)} - 1} r_m m. \quad (5)$$

A mortgage is terminated prior to maturity if the borrower prepays it in full, refinances it, or sells their house. In this case, the lender receives the remaining loan balance plus interest. A household who defaults on their loan eliminates their debt outstanding but surrenders their house to the lender. The lender then incurs a foreclosure cost given by $\gamma(q)$. These costs depend on the contract type on which the debtor defaults according to

$$\gamma(q) = \begin{cases} \gamma_L & \text{if } q = L \\ \gamma_H & \text{if } q = H. \end{cases} \quad (6)$$

The parameters $\gamma_L > 0$ and $\gamma_H > 0$ allow the costs to vary across contract types.\textsuperscript{24} This is

\textsuperscript{23}See Boar, Gorea and Midrigan (2020) and Greenwald (2018).

\textsuperscript{24}They represent legal costs associated with judicial foreclosures, administrative overhead, and lack of maintenance that results from the property remaining unoccupied. For example, Campbell, Giglio and Pathak (2011) estimates that, after foreclosure, houses sell at a 27% discount.
because major reforms to underwriting standards since the Great Recession do not explicitly ban certain loan features but, instead, discourage lenders from originating mortgages with those features by exposing them to greater legal liability in the event of borrower default. Any gap in the foreclosure cost parameters represents the differential legal treatment received by loans that do or do not comply with an underwriting standard required by a given policy.

4.2 Household’s optimization problem

The household’s optimization problem is written recursively. The expected discounted lifetime utility of an age- \( j \) renter in state \( \omega \) is

\[
W^R_j(\omega) = \max_{x_j(\omega)} \{ V^R_j(\omega), V^M_j(\omega) \}. \tag{7}
\]

The expected discounted lifetime utility of an age- \( j \) homeowner in state \( \omega \) is

\[
W^O_j(\omega) = \max_{x_j(\omega)} \{ V^R_j(\omega), V^M_j(\omega), V^P_j(\omega), V^D_j(\omega) \}. \tag{8}
\]

The value functions inside the maximum operators of Equations (7) and (8) correspond to discrete choices over housing tenure and loan adjustment available in state \( \omega \). \( V^R_j(\omega) \) is the value of renting; \( V^M_j(\omega) \) is the value of obtaining a new loan; \( V^P_j(\omega) \) is the value of making a payment on an existing loan; and \( V^D_j(\omega) \) is the value of defaulting on outstanding debt. \( x_j(\omega) \in \{ R, M, P, D \} \) denotes the decision to rent, obtain a new loan, continue with an existing loan, and default, respectively. Figure 4 summarizes these choices.

The timing of the model is as follows. At the beginning of the period, a household receives the realizations of their idiosyncratic shocks. They solve the optimization problems associated with available housing tenure and loan adjustment choices and select the option
that yields the highest expected lifetime utility. Consumption occurs at the end of the period.

If renting, a household solves

$$V^R_j(\omega) = \max_{c,s,a} u_j(c,s) + \beta \mathbb{E}_{\delta',p',z'|p,z} W^R_{j+1}(\omega')$$

s.t.

$$c + R ps + a' \leq y_j(z) + (1+r) a + (1-\delta) ph - (1+r_m) m - 1_{h' \neq 0} \kappa_h$$
$$a' \geq 0$$
$$\omega' = (a',0,\delta',p',0,0,0,z').$$

If owning and obtaining a new mortgage, a household solves

$$V^M_j(\omega) = \max_{c,h',m'} u_j(c,h') + \beta \mathbb{E}_{\delta',p',z'|p,z} W^O_{j+1}(\omega')$$

s.t.

$$c + a' + ph' \leq y_j(z) + (1+r) a + (1-\delta) ph - (1+r_m) m + m' - 1_{h' \neq h} \kappa_h - 1_{m' > 0} \kappa_m$$
$$m' \leq \theta ph'$$
$$\pi_{\min,j}(m',r'_{m,j}(\omega)) \leq \lambda_j(\omega) y_j(z)$$
$$a' \geq 0$$
$$\omega' = (a',h',\delta',p',m',q'_{j}(\omega),r'_{m,j}(\omega),z').$$

where the tightness of the DTI limit $\lambda_j(\omega)$ and the contract type $q'_{j}(\omega)$ are determined by Equations (3) and (4), respectively. In this problem, I make explicit the interaction of households with the mortgage market when they enter into a new loan. Its interest rate $r'_{m,j}(\omega)$ is determined in equilibrium and carried into tomorrow’s state $\omega'$ because it is fixed for the duration of the mortgage. Likewise, the contract type implied by the household’s loan characteristics $q'$ appears in $\omega'$ because it affects the lender’s payoff in the event of future default. The endogenous default risk used to qualify a household for a high-DTI loan is the probability that an age-$j$ household who obtains a new loan in state $\omega$ defaults at age $j+1$,

$$\psi_j(\omega) \equiv \mathbb{E}_{\delta',p',z'|p,z} \left\{ 1_{x_{j+1}(\omega') = D} | x_j(\omega) = M \right\} .$$

\[25\]This problem nests the case of households adjusting their house size without obtaining a new loan. Such households account for 0.2% of households in the stationary distribution in the calibrated model.
If continuing with an existing loan, an owner solves

$$V_j^P(\omega) = \max_{c,a',m'} u_j(c,h) + \beta \mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^{Q}(\omega')$$

s.t.

$$c + \delta ph + a' \leq y_j(z) + (1 + r) a - (1 + r_m) m + m'$$
$$m' \leq (1 + r_m) m - \pi_{\min,j}(m,r_m)$$
$$a' \geq 0$$
$$\omega' = (a', h, \delta', p', m', q, r_m, z').$$

(12)

In this case, the household’s state in the next period reflects the fact that they continue with their predetermined house size, contract type, and mortgage interest rate.

If defaulting, an owner with outstanding debt solves

$$V_j^D(\omega) = \max_{c,s,a'} u_j(c,s) - \xi + \beta \mathbb{E}_{\delta',p',z'|p,z} [\varphi W_{j+1}^{R}(\omega') + (1 - \varphi) V_{j+1}^{R}(\omega')]$$

s.t.

$$c + Rps + a' \leq y_j(z) + (1 + r) a$$
$$a' \geq 0$$
$$\omega' = (a', 0, \delta', p', 0, 0, 0, z').$$

(13)

Because a defaulting household discharges existing debt and loses their house, they do not cover the minimum mortgage payment or housing maintenance costs that are otherwise due. They incur a flow utility loss \(\xi > 0\) and are excluded from owning a house and borrowing for a stochastic period of time. With exogenous probability \(\varphi\), they regain access to these markets in the next period and, with probability \(1 - \varphi\), they must continue renting.

At age \(T\), the household cannot borrow further. This imposes the restriction that \(m' = 0\) on the optimization problems associated with obtaining a new loan and continuing with an existing mortgage. End-of-life wealth consists of liquid savings and, if applicable, the value of the home net of depreciation, \(W_{T+1} = (1 + r) a' + (1 - \delta') p'h'.\)

### 4.3 Financial intermediary’s optimization problem

Mortgage contracts are issued by financial intermediaries who maximize expected profit. Intermediaries are risk neutral, infinitely lived, and perfectly competitive. They have perfect information and fully observe a household’s current state and decision rules. They discount the future at rate \(r + \phi\), where \(\phi > 0\) is a parameter that captures mortgage servicing costs.
The present value of an existing mortgage held by an age-\(j\) household in state \(\omega\) is

\[
\Pi_j(\omega) = \begin{cases} 
(1 + r_m) m & \text{if repay} \\
(1 - \delta) ph - \gamma(q) & \text{if default} \\
(1 + r_m) m - m'_j(\omega) + \frac{1}{1 + r + \phi} \mathbb{E}_{\delta',p',z'|p,z} \Pi_{j+1}(\omega') & \text{otherwise},
\end{cases}
\]

where \(\omega' = (a'_j(\omega), h'_j(\omega), \delta', p', m'_j(\omega), q'_j(\omega), r'_{m,j}(\omega), z')\) reflects the household’s optimal choices. If the borrower repays the loan, then the lender receives the remaining balance plus interest. If they default, then the lender recovers the value of the house posted as collateral net of depreciation and foreclosure costs. If they continue with the loan, then the lender receives their mortgage payment and the continuation value of the loan. On a loan-by-loan basis, the face value of a newly originated loan equals the expected present value of its future cash flows in equilibrium. Thus, the lender offers an interest rate \(r'_{m,j}(\omega)\) that satisfies the zero-expected profit condition

\[
m'_j(\omega) = \frac{1}{1 + r + \phi} \mathbb{E}_{\delta',p',z'|p,z} \Pi_{j+1}(\omega').
\]

5 Calibration

The goal of the calibration is to ensure that the stationary distribution of households in the model matches both salient features of household balance sheets and mortgage market outcomes under the current debt payment-to-income limit documented in Section 3. This makes the model a good representation of the institutional status quo and an appropriate setting for policy evaluation. The model is in part calibrated to match the observed shares of borrowers above and just below the 45% DTI limit. However, the calibration does not target the observed discontinuity in default risk at the limit or the fraction of creditworthy borrowers on the limit. I instead use these outcomes to validate the mechanism in the model.

Where possible, I assign parameter values directly by relying on external empirical evidence or standard values in the literature. I calibrate the remaining parameters internally by minimizing the weighted distance between an array of empirical moments and their model equivalents. I discuss the two sets of parameters separately, then show that the calibrated model does well in matching life-cycle profiles of household balance sheets and consumption, along with aggregate moments regarding homeownership, mortgage debt, and default.

5.1 Externally calibrated parameters

Table 1 lists the directly assigned parameters and their sources.
Table 1. Externally assigned parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_j$ Age-specific income</td>
<td></td>
<td>PSID (1999–2017)</td>
</tr>
<tr>
<td>$\mu_1$ Distribution of age-1 households</td>
<td></td>
<td>SCF (2016)</td>
</tr>
<tr>
<td>$\Phi$ Pension income</td>
<td></td>
<td>Guvenen and Smith (2014)</td>
</tr>
<tr>
<td>$\varsigma_j$ DTI constraint offset</td>
<td></td>
<td>SCF (2016) and AHS (2017)</td>
</tr>
<tr>
<td>$\delta$ Low depreciation rate (%)</td>
<td>1.064</td>
<td>PSID (2005–2017)</td>
</tr>
<tr>
<td>$\lambda$ Statutory DTI limit (%)</td>
<td>45</td>
<td>Freddie Mac</td>
</tr>
<tr>
<td>$\varphi$ Exclusion from mortgage market</td>
<td>0.143</td>
<td>Experian</td>
</tr>
<tr>
<td>$r$ Risk-free rate (%)</td>
<td>1.270</td>
<td>FRED (1971–2016)</td>
</tr>
<tr>
<td>$\rho_p$ Persistence of house price shock</td>
<td>0.970</td>
<td>Mitman (2016)</td>
</tr>
<tr>
<td>$\rho_z$ Persistence of income shock</td>
<td>0.977</td>
<td>Storesletten, Telmer and Yaron (2004)</td>
</tr>
<tr>
<td>$\sigma$ Risk aversion</td>
<td>2</td>
<td>Standard in literature</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$ Std. dev. of income shock</td>
<td>0.155</td>
<td>Storesletten, Telmer and Yaron (2004)</td>
</tr>
<tr>
<td>$\sigma_\eta$ Std. dev. of house price shock</td>
<td>0.080</td>
<td>Mitman (2016)</td>
</tr>
<tr>
<td>$T$ Number of model periods</td>
<td>56</td>
<td>U.S. life expectancy of age 80</td>
</tr>
<tr>
<td>$T_R$ Retirement age</td>
<td>41</td>
<td>Retirement at age 65</td>
</tr>
<tr>
<td>$\theta$ Statutory LTV limit (%)</td>
<td>85</td>
<td>Greenwald (2018)</td>
</tr>
<tr>
<td>$\vartheta$ Inv. elasticity of sub. btw. $c$ and $s$</td>
<td>0.800</td>
<td>Piazzesi, Schneider and Tuzel (2007)</td>
</tr>
</tbody>
</table>

Preferences and endowments

One period in the model is one year. Households enter the model at age 25, retire at age 65, and die at age 80, implying $T = 56$ and $T_R = 41$. 26 I rely on estimates from Piazzesi, Schneider and Tuzel (2007) to set the inverse elasticity of substitution between nondurable consumption and housing services $\vartheta$ equal to 0.800. The coefficient of relative risk aversion $\sigma$ is 2, a standard value in the macroeconomics literature.

Following Storesletten, Telmer and Yaron (2004), I set the persistence of shocks to the idiosyncratic component of income $\rho_z$ to 0.977 and its standard deviation $\sigma_\varepsilon$ to 0.155. Following Kaplan and Violante (2014), I use data from the 1999–2017 waves of the Panel Study of Income Dynamics (PSID) to estimate the deterministic age-dependent component of income. I parameterize pension income function following Guvenen and Smith (2014). 27

Liquid savings

The risk-free rate $r$ is set to 1.27%, the difference between the 1-year Treasury constant maturity rate and annual CPI inflation averaged over the years 1971–2016.

26 As in Berger et al. (2017), I start households at age 25 to avoid complications from schooling decisions.
27 I provide additional details on the estimation of the income process in Appendix B.2.
Housing

Per Mitman (2016), the persistence of the idiosyncratic house price shock $\rho_p$ is 0.970 and its standard deviation $\sigma_p$ is 0.080. The low depreciation rate $\delta$ is 1.06% to match the mean ratio of annual home maintenance expenditures to home value in the PSID.28

Mortgages

The statutory DTI limit $\lambda$ is 45%. I parameterize the DTI offset function $\varsigma_j(\omega)$ by regressing household balance sheet and demographic variables on non-mortgage DTI ratios computed from the 2016 Survey of Consumer Finances (SCF). I use the estimated coefficients to project non-mortgage DTI ratios for agents in the model conditional their age and current state. Separately, I use the 2017 American Housing Survey to calculate the mean ratio of housing expenses relative to income. A household’s DTI offset is the sum of their projected non-mortgage DTI ratio and the mean housing expense-to-income ratio.29

The probability of mortgage market re-entry after default $\varphi$ is 0.143. This matches the 7-year period for which a foreclosure flag remains on a consumer’s credit report.30 Following Greenwald (2018), the statutory LTV limit $\theta$ is 85%.

Distribution of age-1 households

To initialize the stationary distribution of households, I stratify a sample of households with heads between ages 23–27 from the 2016 SCF into $N_z$ groups according to their incomes to match the invariant distribution of the income shock $z$, where $N_z$ is the number of discretized income states in the numerical solution of the model. For households in each $z$, I calculate the homeownership rate, the fraction of homeowners with mortgages, mean liquid assets, mean home equity, and the mean interest rate on the first mortgage. I then distribute households across states to match balance sheet statistics from the SCF, as well as the invariant distributions of the depreciation shock $\delta$ and the house price shock $p$.

5.2 Internally calibrated parameters

The remaining 15 parameters of the model are chosen by minimizing the weighted distance between an equal number of empirical targets and their equivalents in the stationary distribution of the model. The targets fall into three broad categories. I calculate moments

28Specifically, I use question F87, “How much did you (and your family living there) spend altogether in [previous year] on home repairs and maintenance, including materials plus any costs for hiring a professional?”
29I describe the estimation of the DTI offset in Appendix B.3.
relevant to mortgage market outcomes from the Freddie Mac Single Family Loan-Level Dataset.\textsuperscript{31} I calculate cross-sectional and life-cycle moments pertaining to household balance sheets from the 2016 SCF.\textsuperscript{32} I take two moments, the mean foreclosure and housing depreciation rates, from aggregate data sources.

Preferences and endowments

The calibration assigns a lower utility weight on nondurable consumption $\alpha_j$ to working relative to retired households (0.709 versus 0.862, respectively). This helps the model match the ratio of aggregate housing wealth to income. The subjective discount factor $\beta$ is 0.907, similar to other values found in the literature, and is informative about the ratio of aggregate net worth to income. The weight on the bequest motive $B$ is 48.825 and limits declines in net worth late in life. The flow utility loss from default $\xi$ is 19.382.

Housing

Three parameters govern the distribution of owner-occupied house sizes: the minimum house size $h$, the ratio between the maximum and minimum house sizes $h_{gap}$, and the spacing parameter for the housing grid $h_{skew}$.\textsuperscript{33} The minimum house size is set to 7.319, the largest house size is 1.203 times larger, and the spacing parameter is 1.977. This implies a slightly right-skewed distribution of house sizes. Following Kaplan, Mitman and Violante (2020), these parameters are pinned down by the 25th and 75th percentiles of the distribution of home equity relative to net worth. This ensures the model is consistent with observed cross-sectional variation in households’ reliance on home equity for wealth accumulation.

The large depreciation shock $\bar{\delta}$ is set to 35.15%. Negative equity is a necessary but not sufficient condition for default in this class of models. Because the LTV constraint prevents households from beginning their tenure as owners with negative equity and house prices in the model are mean reverting, a large depreciation shock acts as a direct shock to expenditures that makes default optimal for some underwater owners.\textsuperscript{34} The probability of a large depreciation shock $\zeta$ is chosen such that the expected value of the depreciation shock is 2.27%.\textsuperscript{35} Its value of 0.035 implies a large depreciation shock occurs once every 27 years. The housing adjustment cost $\kappa_h$ is 0.610, and the rent-price ratio $R$ is 2.430.

\textsuperscript{31}To be consistent with the SCF, I calculate these moments from loans in my sample originated in 2016.
\textsuperscript{32}I describe how these moments are calculated in Appendix B.4.
\textsuperscript{33}Letting $N_h$ be the number of housing grid points and $\bar{h} \equiv h_{gap} \bar{h}$, the value of the $i$th grid point is $h_i = ((i - 1) / (N_h - 1))^{h_{skew}} (\bar{h} - \bar{h}) + \bar{h}$.
\textsuperscript{34}This is the “double trigger” hypothesis of default—see, e.g., Foote, Girardi and Willen (2008). In the stationary distribution of the calibrated model, 63% of households who default experience a large depreciation shock. On average, the DTI ratio at the time of default is 35%.
\textsuperscript{35}See https://apps.bea.gov/scb/account_articles/national/0597niw/tablea.htm.
Mortgages

The default risk threshold for a high-DTI loan $\Psi$ is set to 0.025%. I set the cost of default on a low-DTI loan to lenders $\gamma_L$ equal to the calibrated value of the housing adjustment cost because, in reality, lenders are responsible for selling properties repossessed after a foreclosure. High-DTI loans are not subject to any additional legal liability relative to low-DTI loans in the event of default under current regulations, so I set $\gamma_H = \gamma_L$. As a result, the distinction between loan contract types is not meaningful in the baseline model, but the policy counterfactual in Section 7 will require that $\gamma_L \neq \gamma_H$.

The loan adjustment cost $\kappa_m$ is 0.261. Together with the housing adjustment cost, it implies that an agent who finances a new house with a loan pays around 10% of their home value in transaction costs.\footnote{Berger and Vavra (2015) estimate that the fraction of the value of durable goods lost to adjustment costs is 5.3%. Their model features one durable asset, whereas my model has both illiquid housing and mortgage debt. The fixed cost of obtaining a new loan in Boar, Gorea and Midrigan (2020) is 2.3% of mean per-capita income, but their model also features an i.i.d. utility loss from refinancing. Keys, Pope and Pope (2016) document that many homeowners do not refinance when theory predicts they should, so large transaction costs are needed to rationalize observed refinancing behavior.}

The mortgage servicing cost $\phi$ of 1.29% is pinned down by the observed spread between the risk-free rate and mean real mortgage interest rate.

5.3 Model fit

Table 2 lists internally calibrated parameters, the empirical targets, and their model equivalents. Overall, the model does well at matching the targeted moments, including aggregate foreclosure and homeownership. Crucially, it matches the share of borrowers with DTI ratios above 45%, the share of borrowers with DTI ratios between 43% and 45%, and the mean DTI ratio at origination—key features of the empirical distribution of DTI ratios.

The share of mortgages with a DTI ratio above 45% is highly informative about the parameterization of the default risk threshold $\Psi$. To place its strictness in perspective, 59% of borrowers who obtain new loans in the stationary distribution of the model have a probability of default less than the threshold value.\footnote{In the data, 60% borrowers have a credit score of at least 740, which, as discussed in Section 3.3, is the nearest observable representation of the creditworthiness standard applied by lenders.}

The calibration implies that the pool of borrowers who are creditworthy enough to benefit from lender discretion is fairly large but that a nontrivial share of households is still screened out of high-DTI loans.

The minimum house size is important for matching the observed share of borrowers bunched at the DTI limit. Intuitively, increasing it makes loan balances and DTI ratios larger, but this effect is non-monotonic. If the minimum house size—and, by extension, the required down payment—becomes too large, then some marginal homeowners will instead
Table 2. Internally calibrated parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{j&lt;T_R}$ Working HH nondur. cons. weight</td>
<td>0.709</td>
<td>Aggregate housing stock</td>
<td>2.57</td>
<td>2.54</td>
</tr>
<tr>
<td>$\alpha_{j\geq T_R}$ Retired HH nondur. cons. weight</td>
<td>0.862</td>
<td>DTI ratio at origination</td>
<td>0.35</td>
<td>0.38</td>
</tr>
<tr>
<td>$B$ Bequest weight</td>
<td>48.825</td>
<td>Ret. to working HH net worth</td>
<td>1.94</td>
<td>1.90</td>
</tr>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.907</td>
<td>Aggregate net worth</td>
<td>2.64</td>
<td>2.65</td>
</tr>
<tr>
<td>$\delta$ High depreciation rate</td>
<td>0.352</td>
<td>Mean LTV ratio</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>$h_{gap}/\bar{h}$ Housing grid skewness</td>
<td>1.203</td>
<td>75th pctl. home eq. share</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td>$h_{skew}$ % loans with DTI $\in (43, 45]$</td>
<td>1.977</td>
<td>10.24</td>
<td>11.46</td>
<td></td>
</tr>
<tr>
<td>$\bar{h}$ Smallest house size</td>
<td>7.319</td>
<td>25th pctl. home eq. share</td>
<td>0.47</td>
<td>0.32</td>
</tr>
<tr>
<td>$\kappa_h$ Housing transaction cost</td>
<td>0.610</td>
<td>Homeownershiop rate</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$\kappa_m$ Loan transaction cost</td>
<td>0.261</td>
<td>Share of owners with debt</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>$\phi$ Loan servicing cost (%)</td>
<td>1.286</td>
<td>Mean mort. interest rate (%)</td>
<td>2.62</td>
<td>2.60</td>
</tr>
<tr>
<td>$\Psi$ Default risk threshold (%)</td>
<td>0.025</td>
<td>% loans with DTI $&gt; 45$</td>
<td>8.71</td>
<td>9.61</td>
</tr>
<tr>
<td>$R$ Rent-price ratio</td>
<td>2.430</td>
<td>Aggregate liquid wealth</td>
<td>0.98</td>
<td>1.01</td>
</tr>
<tr>
<td>$\xi$ Utility loss from def.</td>
<td>19.382</td>
<td>Foreclosure rate (%)</td>
<td>0.70</td>
<td>0.66</td>
</tr>
<tr>
<td>$\zeta$ High depreciation shock prob. (%)</td>
<td>3.549</td>
<td>Mean depreciation rate (%)</td>
<td>2.27</td>
<td>2.27</td>
</tr>
<tr>
<td>$\gamma_L$ Low-DTI loan def. cost</td>
<td>0.610</td>
<td>Set to housing adjustment cost $\kappa_h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_H$ High-DTI loan def. cost</td>
<td>0.610</td>
<td>Set to $\gamma_L$ in baseline calibration</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Aggregate housing stock, net worth, and liquid wealth are relative to mean income in the stationary distribution. LTV ratio is outstanding debt relative to home value conditional on owning a home in the cross section, not at loan origination. Home equity share is home equity relative to net worth. $\bar{h}$ is the largest house size in the model. $\gamma_L$ and $\gamma_H$ are not involved in computing the calibration loss function but depend on the housing adjustment cost, which is.

find it optimal to rent. The remaining population of homeowners has higher incomes and net worth so that, in equilibrium, loan balances and DTI ratios at origination decline.

The relatively lower weight on housing services in the utility function of retired households helps the model match the mean DTI ratio at origination. Assuming that mortgage maturity is decreasing in age and that all debt must be repaid at the end of life can mechanically generate very high DTI ratios. All else equal, retired households with a relatively high preference weight on housing services take out larger loans that are amortized over a relatively short remaining lifetime, implying large payments each period. Retired households also have lower incomes. Reducing retired households’ utility weight on housing services prevents the model from generating counterfactually large DTI ratios.38

As shown in Figure 5, the model captures important cross-sectional and life-cycle dimensions of household income, consumption, and savings. Parameters governing the income process in the model are directly assigned, so it is unsurprising that the model

38This aspect of the model can be modified by making loan maturity independent of age, but doing so requires tracking another state variable.
Figure 5. Household consumption, income, and balance sheets over the life cycle

Notes: Nondurable consumption and income are both normalized relative to households with heads between the ages 25–32. LTV ratio is outstanding mortgage debt scaled by value of the primary residence conditional on owning a home. Debtor share is the fraction of homeowners with positive mortgage debt. Data source: 2016 SCF for household income and net worth, 2017 PSID for nondurable expenditures.

reproduces life-cycle income profiles.\textsuperscript{39} The calibration does not target any moments related to consumption, however. With the exception of the net worth ratio of retired to working households, life-cycle patterns of household wealth are also not explicitly targeted in the calibration. The model matches well the increase in and composition of net worth over the life cycle. It is also broadly consistent with homeownership rates and the extensive margin of debt as households age. In the model, homeownership flattens out in retirement but continues rising in the data. This stems from a tension in the calibration arising from the lower weight on housing services in the utility function of retired agents. The quantitative trade-off is a slight counterfactual decrease in homeownership at the end of life.

\textsuperscript{39}Income life-cycle profiles in the data and model do not match exactly because I use the SCF in Figure 5 but the PSID to parameterize the income process in the model.
6 Quantitative results of the model

Having validated that the calibrated model matches the empirical distribution of borrowers’ debt payment-to-income ratios and the consumption and savings behavior of households over the life cycle, I now establish that it successfully captures the novel stylized facts documented in Section 3. The model quantitatively replicates the discontinuity in default risk observed at the 45% debt payment-to-income limit. I show that the loan screening technology in the baseline model is needed to explain this: a standard model without it counterfactually predicts that default risk is strictly increasing in the DTI ratio and overstates the dispersion of default risk across borrowers.

The calibrated model reveals why an identical DTI limit binds for households in different regions of the state space. I show that whether a household is constrained by the limit along the intensive or extensive margin of mortgage debt is informative about this heterogeneity. Agents constrained along the extensive margin are current renters transitioning to homeownership who exhibit the highest propensity to default. Their exit from the mortgage market in response to the DTI requirement reduces the overall variance in default risk among borrowers obtaining new loans. By contrast, agents constrained along the intensive margin are existing homeowners who refinance their loans in order to relax current and future liquidity constraints. Their high level of home equity makes default unlikely. Their switch from a high- to low-DTI loan accounts for the sharp discontinuity in default risk observed at the current 45% cutoff. Both groups of households have high desired DTI ratios but vary substantially in their ability to repay, indicating that the DTI ratio itself is an insufficient proxy for creditworthiness.

6.1 Borrower selection into high-DTI loans

The quantitative success of the model rests on its ability to yield the observed difference in default risk of borrowers on either side of the debt payment-to-income limit. The calibration implies that only sufficiently safe households qualify for high-DTI loans but leaves open the question of precisely how much more creditworthy borrowers above the limit are relative to those below it. It also places no explicit restrictions on the characteristics of borrowers just below the 45% cutoff in the stationary distribution of the model.

The model does well in this regard, generating a discontinuity in interest rates at the statutory DTI limit in line with the empirical estimates from Section 3.2. To show this, I plot in the left-hand panel of Figure 6 mortgage interest rates as a function of DTI ratios at origination in the stationary distribution of the model and in the data. Because the model is parameterized to match the residential mortgage market under current underwriting
standards, the relevant empirical benchmark is mortgages originated since 2009. In the model, the interest rates paid by borrowers with DTI ratios between 43% and 45% is 13 basis points greater than those above the limit, compared to 6.7 in the data. The equilibrium cost of borrowing associated with high-DTI loans is 2.3 basis points below average, compared to 1.4 in the data. Therefore, the model is quantitatively consistent with the non-monotonic relationship between default risk and DTI ratios found in the data.

The loan screening technology is essential for generating this relationship between default risk and DTI ratios in the model. The right-hand panel of Figure 6 plots the equilibrium relationship between interest rates and DTI ratios in a version of the model in which the default risk threshold for high-DTI loans is entirely relaxed and all other parameters in the model are unchanged. This collapses my model to the standard theoretical framework in which households face only a loan-to-value constraint when obtaining a new loan. The relevant empirical benchmark in this case is mortgages originated prior to the introduction of the current DTI limit in 2009. The standard framework is unable to generate non-monotonicity in the relationship between borrowers’ default risk and DTI ratios. Instead, default risk is strictly increasing in the DTI ratio at origination, qualitatively replicating the

Because the foreclosure costs of a low- and high-DTI loan remain identical, the expected profit of a loan is independent of contract type. I fully relax the default risk threshold for high-DTI loans by setting $\Psi = 1$ so that it never binds. This means that high-DTI loans are screened no more than low-DTI loans. Therefore, there is no distinction between the two in this counterfactual parameterization of the model.
correlation that existed when mortgage underwriting standards did not include a binding DTI limit. This version of the model over-predicts the variance of default risk associated with mortgage originations: relative to the data, too many households with a high propensity to default in their current state enter into new loans. I speculate that, in reality, mortgage lenders performed some informal screening of their loan applicants that limited the overall riskiness of originations even in the absence of a formal DTI requirement in the pre-policy period.\footnote{To use one example, Freddie Mac’s selling guidelines have long required additional justification from a lender when originating a mortgage with a DTI ratio above 36%.
} This suggests that the standard framework requires some additional friction that curtails the entry of the least creditworthy households into the mortgage market.\footnote{I present additional quantitative results in Appendices B.5 and B.6.}

6.2 Identifying the characteristics of constrained households

Next, I identify the characteristics and allocations of households who are constrained by the current debt payment-to-income limit. In doing so, I explain how the optimal choices of such households rationalize the observed non-linear relationship between default risk and DTI ratios. The advantage of using a structural model is that I can compute households’ counterfactual loan choices, including those would not be observed in any data on mortgage originations. As a result, I define a constrained household as an agent who would optimally choose a high-DTI loan but has an endogenous default risk above the qualifying threshold that makes it infeasible.

I show that the current DTI limit binds not just for borrowers observed at the 45% cutoff in the data; indeed, adjustments along both the intensive and extensive margins of debt are needed to account for the discontinuity in default risk at the DTI limit. 53% of constrained households adjust along the extensive margin. These households, who switch from obtaining a new high-DTI mortgage to not obtaining any loan, are the riskiest borrowers in a no-DTI limit setting, and their exit from the mortgage market lowers the dispersion in default risk among remaining borrowers. 47% of constrained households adjust along the intensive margin by switching from a high- to a low-DTI loan, decreasing their loan size until their DTI ratio falls below the statutory limit. The shift of these borrowers within the distribution of DTI ratios accounts for the difference in default risk around the limit.

The importance of adjustments along the extensive margin of mortgage debt is corroborated by existing empirical evidence. For instance, DeFusco, Johnson and Mondragon (2020) document that, in response to the introduction of a DTI limit in the jumbo loan market, the decrease in high-DTI loans is larger than what existing estimates of the elasticity of mortgage demand to interest rates would imply. They argue that the large quantity

\footnote{To use one example, Freddie Mac’s selling guidelines have long required additional justification from a lender when originating a mortgage with a DTI ratio above 36%.
}
response reflects changes in credit supply. The screening technology in my model generates a similar outcome whereby loans deemed too risky are simply not originated in equilibrium.

To better understand the trade-offs that confront constrained households, I first study the characteristics of high-DTI borrowers in the model. As seen in Table 3, virtually all high-DTI borrowers are homeowners refinancing their loans. The high-DTI option is feasible not only due to their high net worth but also on account of their substantial home equity.\footnote{Mean net worth to income for high-DTI borrowers is 4.50, compared to 2.65 for all households.} For plausibly calibrated adjustment costs, such an owner will not default because they can exercise the option to sell their house. Nevertheless, a high-DTI loan is optimal for these households because they value the additional liquidity from refinancing for precautionary reasons. Compared to a low-DTI loan, a high-DTI loan allows this agent to increase their liquid savings considerably.\footnote{The higher default risk associated with high-DTI borrowers’ counterfactual low-DTI loans reflects non-negativity constraint on consumption possibly binding in the future after housing-related expenses are met.}

Households constrained along the intensive margin by the DTI limit have similar incomes to high-DTI borrowers but considerably less wealth.\footnote{Conditional on being an existing owner, mean home equity of borrowers at the limit is 1.98 units of the numeraire. This is still much lower than the mean home equity of high-DTI borrowers of 2.74.} These constrained agents face smaller streams of required housing outlays, however. Because their future expected liquidity constraints are less binding, they can optimally substitute into low-DTI loans with slightly smaller balances at origination. These households account around one-third of all borrowers found at the statutory limit. Because borrowers bunched below the limit are a mix of constrained households and agents for whom a DTI ratio near 45% was always optimal, the model can match the meaningful heterogeneity in default risk observed at the DTI limit.

Households constrained along the extensive margin differ significantly in their underlying characteristics. More than half are existing renters, indicating that the high-DTI loans they would have chosen are purchase loans.\footnote{Only 0.8% of households who obtain new loans with DTI ratios below 43% are constrained. No constrained households are found among owners who continue with an existing loan, households who adjust their house size without simultaneously obtaining a mortgage, or those who default.} Their highly liquid wealth portfolio corroborates this, as a large stock of liquid assets is needed to cover the down payment and adjustment costs associated with home buying.\footnote{Households constrained by the DTI limit on the extensive margin have a liquid wealth to income ratio of 2.13, compared to 1.01 for the entire population.} The indivisibility of owner-occupied housing, captured by the minimum house size, is crucial for this. It induces lumpy adjustment to housing by effectively bounding the face value of a purchase loan from below. A smaller DTI-constrained loan is infeasible for agents who need a mortgage for this reason, as shown by the negative consumption it implies. As a result, these households find it optimal to forgo homeownership and continue renting. Among all debtors, first-time homeowners are most likely to default
Table 3. Characteristics and loan choices of constrained households in the model

<table>
<thead>
<tr>
<th></th>
<th>DTI &gt; 45</th>
<th>DTI ∈ (43, 45]</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>1.15</td>
<td>1.06</td>
<td>1.04</td>
</tr>
<tr>
<td>Liquid wealth</td>
<td>2.43</td>
<td>1.27</td>
<td>2.21</td>
</tr>
<tr>
<td>Home equity</td>
<td>2.74</td>
<td>1.74</td>
<td>0.70</td>
</tr>
<tr>
<td>Net worth</td>
<td>5.17</td>
<td>3.01</td>
<td>2.91</td>
</tr>
<tr>
<td>Owner share</td>
<td>0.98</td>
<td>0.88</td>
<td>0.45</td>
</tr>
<tr>
<td>Min. loan payment</td>
<td>0.59</td>
<td>0.24</td>
<td>0.30</td>
</tr>
<tr>
<td>Maintenance costs</td>
<td>0.15</td>
<td>0.13</td>
<td>0.28</td>
</tr>
<tr>
<td>Income shock</td>
<td>0.85</td>
<td>0.60</td>
<td>0.64</td>
</tr>
<tr>
<td>House price shock</td>
<td>1.06</td>
<td>0.89</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Low-DTI loan

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan size</td>
<td>3.11</td>
<td>4.50</td>
</tr>
<tr>
<td>Int. rate (%)</td>
<td>2.82</td>
<td>2.68</td>
</tr>
<tr>
<td>Def. prob. (%)</td>
<td>16.32</td>
<td>1.24</td>
</tr>
<tr>
<td>Liq. savings</td>
<td>0.72</td>
<td>0.92</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.49</td>
<td>0.73</td>
</tr>
</tbody>
</table>

High-DTI loan

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan size</td>
<td>5.62</td>
<td>5.21</td>
</tr>
<tr>
<td>Int. rate (%)</td>
<td>2.58</td>
<td>2.71</td>
</tr>
<tr>
<td>Def. prob. (%)</td>
<td>0.00</td>
<td>1.19</td>
</tr>
<tr>
<td>Liq. savings</td>
<td>2.99</td>
<td>1.60</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.69</td>
<td>0.75</td>
</tr>
</tbody>
</table>

% of households         | 3.19     | 1.23          | 1.36 |

Notes: “DTI > 45” are agents who choose high-DTI loans. "DTI ∈ (43, 45]” are constrained agents who choose a loan with DTI ratios between 43% and 45%. “Rent” are constrained agents who rent. Income, balance sheet, and expenditure variables are in units of the numeraire. Owner share is the fraction of existing homeowners. The bottom half of the table displays optimal choices conditional on a low- or high-DTI loan. “% of households” is the group’s share in the total population of the model.

because they begin their tenure as owners with low equity in their homes. These leaves them vulnerable to adverse shocks that can make default optimal.

7 Evaluating the Dodd-Frank ability-to-repay rule

Having established that the calibrated model accounts for the observed heterogeneity in default risk across borrowers conditional on their debt payment-to-income ratios, I use it to study the aggregate and distributional effects of two proposed versions of the Dodd-Frank Act ability-to-repay rule. Its goal is to ensure lenders make a “reasonable and good faith determination” of the borrower’s ability to repay a mortgage and exposes a lender who
originates a “risky” loan to greater legal liability if the debtor eventually defaults on it.\(^{48}\) The two versions of the rule differ in their use of a DTI- versus a price-based definition of a risky loan. There has been significant disagreement over the rule’s effects, and it is not \textit{ex ante} obvious which approach is more beneficial for households.

The counterfactual exercise compares the steady state of the model under the two versions of the ability-to-repay rule. Relative to the original rule, the revised rule raises aggregate welfare by 0.4% in consumption-equivalent terms, with 95% of households preferring it. Welfare gains are concentrated among households whose mortgages would incur the DTI-based penalty but not the price-based penalty. Welfare gains are particularly large for renters who are constrained along the extensive margin of debt by the original rule. Under the revised rule, they obtain new loans and become homeowners. From the household’s perspective, the trade-off between the two policies hinges on the correlation between their ability to repay and the “risky” loan feature. Intuitively, because a borrower’s DTI ratio at origination is a poorer proxy of default risk than their interest rate, the original rule raises the price of credit for borrowers who are not especially default-prone in their current state.

### 7.1 Description of the Dodd-Frank ability-to-repay rule

To implement the ability-to-repay rule, the Consumer Financial Protection Bureau (CFPB) defined a category of loans called qualified mortgages (QMs) that are assumed to comply with it. Originating a mortgage that fails to meet theQM definition subjects a lender to greater legal liability if the borrower eventually defaults. The ability-to-repay rule overlays existing GSE underwriting standards, and responsibility for compliance lies with the lender. Issued in 2014, the \textit{original QM rule} stated that a back-end DTI ratio of 43% or lower was sufficient for a loan to satisfy ability-to-repay rule but exempted mortgages eligible for GSE purchase; this “GSE patch” was scheduled to expire in January 2021. Before it did, the CFPB issued a \textit{revised QM rule} in 2020 that replaced the DTI criterion with one based on the loan’s price.\(^{49}\) A loan meets the revised QM definition if the spread between its annual interest rate and the average prime offer rate is less than 1.5 percentage points.\(^{50}\)

The loan contract types in the model allow me to evaluate the effects of the two versions of the ability-to-repay rule on household welfare by parsimoniously tracking the characteristic

---


\(^{50}\)Compliance with the revised rule is delayed until October 2022 due to the COVID-19 pandemic.
of a loan at origination throughout its duration. Let \( q \in \{L, H\} \) denote a qualified and non-qualified mortgage, respectively. Under the original rule, the contract type depends on whether a borrower’s DTI ratio at origination is less or greater than 43%, i.e.,

\[
q'_{j}(\omega) = \begin{cases} 
L & \text{if } \pi_{\min,j} (m', r'_{m,j}(\omega)) \leq (0.43 - \varsigma_j(\omega)) y_j(z) \\
H & \text{if } \pi_{\min,j} (m', r'_{m,j}(\omega)) > (0.43 - \varsigma_j(\omega)) y_j(z).
\end{cases}
\]

Under the revised rule, the qualified mortgage definition depends on a interest rate threshold. Because the CFPB defines the average prime offer rate as the annual percentage rate offered to “highly qualified borrowers,” I use \( r + \phi \)—i.e., the equilibrium interest rate obtained by a household with zero expected probability of default—as its model equivalent.\(^{51}\) In this case,

\[
q'_{j}(\omega) = \begin{cases} 
L & \text{if } r'_{m,j}(\omega) \leq r + \phi + 0.015 \\
H & \text{if } r'_{m,j}(\omega) > r + \phi + 0.015.
\end{cases}
\]

To capture the increased legal liability associated with non-qualified mortgages, I increase the foreclosure cost on non-QM loans so that \( \gamma_H > \gamma_L \). To calibrate \( \gamma_H \), I follow a cost-benefit analysis by the Consumer Financial Protection Bureau (2013) upon its submission of the original rule.\(^{52}\) This procedure yields a value of 2.374 for \( \gamma_H \), which is 4 times larger than the foreclosure cost on a QM loan. Other parameter values in Tables 1 and 2 are unchanged.

### 7.2 Aggregate effects of the Dodd-Frank ability-to-repay rule

Table 4 presents aggregate outcomes in the steady state of the model under the original and revised versions of the ability-to-repay rule. Outcomes under the status quo, which corresponds to the baseline calibration, are provided as a comparison. Under both versions of the rule, the aggregate default rate declines relative to the status quo. Two mechanisms underlie this result. All else equal, the lender passes through the higher cost of non-qualified mortgages to the borrower via a higher interest rate as long as there is some strictly positive probability of default. The higher the household’s likelihood of default, the stronger the pass-through. When the interest rate is high enough, a household reduces their desired loan size. A more subtle effect is that the ability-to-repay rule shrinks the feasible set of loans by driving the profit of originating a loan to zero in states of the world where the expected present value of the contract is already small. Thus, some mortgages that are sustained in

\(^{51}\)See https://www.consumerfinance.gov/ask-cfpb/what-is-a-higher-priced-mortgage-loan-en-1797. The model calibration implies a mortgage must have an interest rate no greater than 4.06% at origination in order to meet the revised rule’s qualified mortgage definition.

\(^{52}\)I describe this in Appendix B.7, where I also show that my results are robust to different values of \( \gamma_H \).
Table 4. Aggregate effects of the ability-to-repay rule

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>Original</th>
<th>Revised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default rate (%)</td>
<td>0.66</td>
<td>0.55</td>
<td>0.60</td>
</tr>
<tr>
<td>Homeownership rate (%)</td>
<td>64.76</td>
<td>62.94</td>
<td>64.07</td>
</tr>
<tr>
<td>Share of owners with mortgage (%)</td>
<td>62.94</td>
<td>61.34</td>
<td>62.36</td>
</tr>
<tr>
<td>LTV ratio (%)</td>
<td>63.33</td>
<td>63.36</td>
<td>63.27</td>
</tr>
<tr>
<td>DTI ratio (%)</td>
<td>38.21</td>
<td>38.08</td>
<td>38.20</td>
</tr>
<tr>
<td>Mortgage interest rate (%)</td>
<td>2.60</td>
<td>2.60</td>
<td>2.59</td>
</tr>
<tr>
<td>Share of mortgages with DTI &gt; 43%</td>
<td>21.07</td>
<td>19.64</td>
<td>21.31</td>
</tr>
<tr>
<td>Share of high-price mortgages (%)</td>
<td>0.08</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>Aggregate net worth</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
</tr>
<tr>
<td>Aggregate liquid wealth</td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
</tr>
<tr>
<td>Aggregate home equity</td>
<td>1.64</td>
<td>1.63</td>
<td>1.64</td>
</tr>
</tbody>
</table>

Notes: “Current,” original,” and ”revised” refer to stationary distribution of the model under the baseline calibration and the two version of the ability-to-repay rule. Loan characteristics are calculated at origination. A high-price mortgage is a non-qualified mortgage under the revised rule. Aggregate net worth, liquid wealth, and home equity are relative to mean income.

equilibrium under the current policy are simply not originated under the Dodd-Frank Act.

The model highlights the trade-off policymakers face between reducing mortgage default on one hand and preserving access to homeownership on the other. The model predicts that the decrease in the foreclosure rate would have been 80% larger under the DTI-based rule than the price-based rule. This is driven by the fact that, in the stationary distribution of the model, around 20% of originations have a DTI ratio of least 43%, whereas only 0.1% are considered high-price loans. In that sense, the original ability-to-repay rule would have been more binding for borrowers, given the large observed share of mortgages that would be directly affected by it. The decrease in foreclosures is accounted for by changes in homeownership and debt along the extensive margin. Conditional on obtaining a new mortgage, the LTV ratio at origination is nearly identical across the two reforms, while the homeownership rate is 1.13 percentage points higher under the revised rule.\textsuperscript{53}

The increase in homeownership accounts for the aggregate welfare gain from the reform in the ability-to-repay rule. In consumption-equivalent terms, the average welfare change is 0.4%.\textsuperscript{54} Under incomplete markets, home equity has self-insurance value despite its illiquidity.\textsuperscript{55} To demonstrate this, Table 5 contains consumption-equivalent welfare changes for households conditional on changes in housing tenure choice when moving from the original

\textsuperscript{53}To put this figure into perspective, the U.S. homeownership rate rose 5 percentage points between 1994–2004. See https://fred.stlouisfed.org/series/RSAHORUSQ156S.

\textsuperscript{54}I define consumption-equivalent welfare change in Appendix B.8.

\textsuperscript{55}In an earlier version of their paper, Boar, Gorea and Midrigan (2020) show that consumption smoothing in a model with one liquid and one illiquid asset is higher than in an identical one with a single liquid asset.
Table 5. Welfare changes by housing tenure changes

<table>
<thead>
<tr>
<th></th>
<th>Share (%)</th>
<th>CEV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own-to-own</td>
<td>63.46</td>
<td>0.10</td>
</tr>
<tr>
<td>Own-to-rent</td>
<td>0.08</td>
<td>-17.36</td>
</tr>
<tr>
<td>Rent-to-own</td>
<td>0.80</td>
<td>21.50</td>
</tr>
<tr>
<td>Rent-to-rent</td>
<td>35.66</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Notes: “Own” and “rent” refer a household’s optimal housing tenure choice in their idiosyncratic state under the original or revised ability-to-repay rule, not their predetermined housing wealth. “CEV” is consumption-equivalent welfare change.

to the revised rule. The highest welfare gains accrue to households in the model who switch from renting to owning, while the highest welfare losses are experienced by households who make the opposite switch. Households who rent under both rules are also better off. Because a new homeowner is on average more likely to have a large DTI ratio than a high interest rate spread, the practical effect of the reform is to relax credit conditions in the economy. This affects renters through their continuation values. Households who own under both versions of the rule experience a small, positive welfare gain. This is driven by the fall in interest rates on loans that would incur the DTI-based penalty but not the price-based one.

In my theoretical environment, I have abstracted from negative externalities that may be associated with default and leverage; my calculations should be considered an upper bound on the welfare effects of the ability-to-repay rule. I have also abstracted from equilibrium house prices in my model. As Kaplan, Mitman and Violante (2020) have shown, the assumptions of long-term mortgage contracts and a rental housing market with complete integration together imply that changes in credit conditions exhibit little pass-through to equilibrium house prices. Because my model shares these two features, incorporating feedback effects through house prices should not change the results substantially.

7.3 Distributional effects of the Dodd-Frank ability-to-repay rule

To illustrate the trade-offs at work in the counterfactual, I focus on agents who have a probability of default high enough that they are subject to the statutory DTI limit. The higher cost of a non-qualified mortgage only lowers the expected value of the loan if a borrower’s optimal choices imply a strictly positive likelihood of default. In the extreme case where a household never defaults, costs imposed by the ability-to-repay rule are irrelevant for loan pricing. Table 6 displays the welfare changes and optimal choices under the original and revised ability-to-repay rule of three groups of agents who meet this criteria.

The first group of agents are those who obtain new loans with a DTI ratio 43% or less
Table 6. Effect of the ability-to-repay rule on households with default risk

<table>
<thead>
<tr>
<th></th>
<th>DTI ≤ 43</th>
<th>DTI ∈ (43, 45]</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Borrow</td>
<td>Rent</td>
<td>Borrow</td>
</tr>
<tr>
<td>Income</td>
<td>1.97</td>
<td>1.14</td>
<td>1.34</td>
</tr>
<tr>
<td>Liquid wealth</td>
<td>0.90</td>
<td>0.71</td>
<td>1.33</td>
</tr>
<tr>
<td>Home equity</td>
<td>0.80</td>
<td>0.06</td>
<td>1.75</td>
</tr>
<tr>
<td>Home equity (if own)</td>
<td>0.98</td>
<td>0.32</td>
<td>2.13</td>
</tr>
<tr>
<td>Net worth</td>
<td>1.69</td>
<td>0.77</td>
<td>3.07</td>
</tr>
<tr>
<td>Owner share</td>
<td>0.81</td>
<td>0.20</td>
<td>0.82</td>
</tr>
</tbody>
</table>

|                  |          |                |        |        |        |        |
| Loan size        | 5.50     | 3.39           | 5.79   | 5.16   | 4.60   | 3.36   |
| Int. rate (%)    | 2.63     | 3.02           | 2.85   | 3.02   | 3.54   | 4.16   |
| Def. prob. (%)   | 0.89     | 2.72           | 1.44   | 2.17   | 46.58  | 74.51  |
| Liquid savings   | 0.96     | 0.23           | 1.09   | 1.12   | 0.29   | 0.04   |
| Consumption      | 1.00     | 0.29           | 0.82   | 4.36   | 0.14   | −0.13  |

|                  |          |                |        |        |        |        |
| Loan size        | 5.49     | 3.30           | 5.76   | 3.45   | 4.94   | 3.49   |
| Int. rate (%)    | 2.62     | 4.15           | 2.71   | 2.71   | 2.95   | 3.96   |
| Def. prob. (%)   | 0.89     | 71.77          | 1.47   | 2.55   | 5.16   | 69.21  |
| Liquid savings   | 0.95     | 0.21           | 1.05   | 1.10   | 0.43   | 0.07   |
| Consumption      | 1.00     | 0.29           | 0.82   | 2.72   | 0.27   | −0.10  |
| CEV (%)          | 0.26     | −16.75         | 0.69   | −1.14  | 21.41  | 0.76   |

Notes: The first row splits agents by their loan choice under the original rule. The second row then splits each group based on their choice under the revised rule. Income, balance sheet, and loan characteristics are in units of the numeraire except where noted. “Owner share” is the share of existing owners in a given group. The bottom half of the table contains loan choices under the two rules. “CEV” is consumption-equivalent welfare change.

under the original rule. These are QM loans under the original rule but potentially become non-QM loans under the revised rule. The loan characteristics of households who obtain a new mortgage under both rules are almost unchanged. Their comparatively high income accounts for their low equilibrium default risk and the minimal pass-through of the non-QM penalty. By contrast, households who rent under the revised rule have characteristics that make them more prone to default. As a result, their mean interest rate rises substantially in equilibrium. The stream of larger minimum mortgage payments this implies makes it optimal for these households to rent under the revised rule. They are worse off as a result.

The second group are agents who choose loans with DTI ratios above 43% under the original rule. For households who still choose a new loan under the revised rule, the reform

---

56High-DTI borrowers also violate the original rule, but, because their default risk is very low, their welfare is relatively unaffected. A model that does not generate the selection of relatively creditworthy borrowers into high-DTI loans will overstate the welfare gains associated with the revised rule.
improves welfare because the pass-through of the non-QM penalty is generally lower under the price-based threshold. This is because DTI ratios are more responsive to changes in loan size than interest rates unless the elasticity of default risk to mortgage debt is already large. The opposite occurs for households who instead rent under the revised rule. Given their interest rates are already high under the original rule, there is strong pass-through of the revised rule’s price-based penalty. In response, these households endogenously demand smaller loans in order to lower their equilibrium borrowing costs. This results in reduced consumption, which ultimately makes it beneficial for these agents to switch to renting.

Finally, the third group are households who rent because the original rule constrains them along the extensive margin of debt. As shown in Section 6.2, these households’ characteristics imply high default risk and high DTI ratios. Consequently, the effect of the non-QM penalty on equilibrium interest rates is particularly strong under the original rule. For households who obtain a new loan under the revised rule, the reform in the ability-to-repay rule expands their feasible set of loans. Their welfare is considerably higher as a result. Households who rent under both policies are also better off because, although their housing tenure choice is unchanged in their current state, they are more likely to become homeowners in the future.

To summarize the distributional effects of the reform in the ability-to-repay rule, Figure 7 plots consumption-equivalent welfare changes by income and net worth quintiles. Households with low incomes but medium levels of net worth consist both of liquidity constrained owners
who can borrow at lower interest rates à la the wealthy hand to mouth in Kaplan and Violante (2014), as well as renters with liquid wealth portfolios who obtain a new loan in order to become owners. Households with the highest incomes and net worth experience small changes to their welfare because their default risk is close to zero and they can already self-insure well against negative shocks. Welfare losses are concentrated among marginal homeowners who switch to renting under the revised ability-to-repay rule.

8 Conclusion

This paper has documented that a statutory limit on debt payment-to-income ratios that has been in place since the Great Recession binds for some but not all households and that, as a result, a high desired DTI ratio is not a necessary condition for a high likelihood of default. To rationalize this finding and conduct counterfactual analyses, I introduced a tractable loan screening technology and mortgage contract types to an otherwise standard structural model of mortgage default. The heterogeneity in default risk across households conditional on their DTI ratios that is generated by the calibrated model cautions against the use of DTI limits as a regulatory tool to lower mortgage default. In a policy counterfactual, I found that a proposed reform to the ability-to-repay rule that penalizes lender who originate high-price loans instead of high-DTI loans increases aggregate welfare because a borrower’s interest rate is a better proxy of their likelihood of default than their DTI ratio.

Looking ahead, there are ways in which the issues raised in this paper could be explored further. Incorporating endogenous house prices or an externality arising from household leverage would permit an analysis of why policymakers may want to implement DTI limits in the first place. Shocks to nominal interest rates are an important driver of mortgage refinancing, which is itself relevant for monetary policy transmission. An environment with an endogenous risk-free rate would be more suitable for studying such questions. Finally, the model developed in this paper could be used to study optimal DTI limits.
References


A Empirical appendix

A.1 Additional figures

As context, Table 7 contains summary statistics for the sample of loans used in the empirical analysis in Section 3. Because there is substantial time-series variation in these loan characteristics, Figure 8 plots the mean DTI ratio, high-DTI share, credit score, and interest rate by year. DTI ratios and the share of mortgages with DTI ratios above 45% fell during the Great Recession before recovering somewhat in recent years. Borrowers in the sample are on average more creditworthy now than they were prior to the Great Recession. Mortgage interest rates have trended downward since the mid-2000s.

Figure 9 displays the evolution of the distribution of DTI ratios from 2005 to 2016. There is little visual indication of a binding DTI constraint at any level during the housing boom. Since 2009, the share of mortgages with DTI ratios above 45% has decreased but constitute around 9% of all originations purchased by Freddie Mac. The share of mortgages with DTI ratios at or just below the statutory limit has increased over time. As of 2016, around 10% of mortgages have a DTI ratio between 43% and 45%.

In Figure 10, I show that the distribution of DTI ratios across mortgages purchased by Fannie Mae—the other GSE involved in the secondary market for conforming mortgages—has undergone changes nearly identical to those found in Figure 9. This has occurred even though the two GSEs possess separate underwriting guidelines. Fannie Mae’s DTI requirement is more lax than that of Freddie Mac: for loans underwritten using Fannie’s proprietary software, the maximum allowable DTI ratio is 50%. However, these data indicate that the relevant statutory limit in the conforming loan market is the more strict requirement of 45% regardless of the buyer of the loan.

In Figure 11, I plot residualized LTV ratios as a function of DTI ratios separately for loans originated before and after the introduction of the DTI limit in 2009. Prior to the policy, the LTV ratio is initially increasing in the DTI ratio before flattening out. After the policy, loans to the right of the DTI limit have smaller LTV ratios compared to loans just at or below the limit. To the extent that smaller LTV ratios are indicative of lower risk of default, this finding further corroborates the selection of more creditworthy borrowers into high-DTI loans during the post-policy period. It also ensures that the high-DTI loans observed in the post-period period are not being driven by large loan balances.

To ensure comparability with the sample of loans purchased by Freddie Mac that I use in my empirical analysis, I only include in this figure loans purchased or guaranteed by Fannie Mae with 30-year terms that are collateralized by owner-occupied housing and have non-missing data on DTI ratios, FICO scores, interest rates, and LTV ratios. This leaves me with around 23 million observations.
### Table 7. Loan-level summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>25th pctile.</th>
<th>50th pctile.</th>
<th>75th pctile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI ratio (%)</td>
<td>34.64</td>
<td>10.86</td>
<td>27</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>Credit score</td>
<td>744.39</td>
<td>50.54</td>
<td>711</td>
<td>755</td>
<td>785</td>
</tr>
<tr>
<td>Interest rate (%)</td>
<td>5.01</td>
<td>1.02</td>
<td>4.13</td>
<td>4.88</td>
<td>5.88</td>
</tr>
<tr>
<td>LTV ratio (%)</td>
<td>72.81</td>
<td>16.49</td>
<td>65</td>
<td>78</td>
<td>80</td>
</tr>
<tr>
<td>Loan amount (000s)</td>
<td>224.16</td>
<td>115.54</td>
<td>135</td>
<td>200</td>
<td>297</td>
</tr>
</tbody>
</table>

*Notes:* The DTI ratio, credit score, LTV ratio, and loan amount at origination are reported as whole numbers in the dataset. Source: Freddie Mac Single Family Loan-Level Dataset.

### Figure 8. Loan characteristics over time

*Notes:* The vertical dashed black line at 2009 indicates the introduction of the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.
Figure 9. Distribution of DTI ratios of loans purchased by Freddie Mac

Notes: Mortgages are grouped into 1-percentage point bins. The dashed vertical line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.
Figure 10. Distribution of DTI ratios of loans purchased by Fannie Mae

Notes: Mortgages are grouped into 1-percentage point bins. The dashed vertical line indicates the 45% DTI limit. Source: Fannie Mae Single-Family Loan Performance Data.
Figure 11. Loan size as a function of the DTI ratio at origination

Notes: The LTV ratio is residualized with respect to a vector of quarter dummy variables. Averages are computed for each 1-percentage point DTI bin for pre- and post-2009Q1 observations separately. The vertical dashed black line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.
A.2 Difference-in-differences regression

To control for other observable variables that may affect the empirical patterns in Section 3.2, I use a difference-in-differences specification common to this literature—e.g., DeFusco, Johnson and Mondragon (2020)—to estimate the change in the characteristics of a high-DTI loan, relative to a low-DTI loan, after the introduction of the 45% DTI limit. The regression equation is

\[ y_{it} = \alpha + \beta_1 HighDTI_i + \beta_2 (HighDTI_i \times Policy_t) + \gamma_t + X'_i \delta + \varepsilon_{it}, \]  

(16)

where \( y_{it} \) is a characteristic of loan \( i \) originated in quarter \( t \), \( \alpha \) is a constant, \( \gamma_t \) is a vector of quarter dummy variables, \( X_i \) is a vector of loan-specific characteristics, and \( \varepsilon_{it} \) is an error term clustered at the state level.\(^{58}\) \( HighDTI_i \) equals 1 if the DTI ratio of loan \( i \) exceeds 45%. \( Policy_t \) equals 1 if the quarter of origination is 2009Q1 or later. The coefficient of interest is \( \beta_2 \), which represents the differential change in the dependent variable for high-DTI loans relative to low-DTI loans after the introduction of the 45% DTI limit. When estimating the regression, I restrict my sample to loans with a DTI ratio between 40% and 50%, and robust standard errors are clustered at the state level.\(^{59}\)

I estimate the regression using the credit score and interest rate as dependent variables. Table 8 displays estimated values for \( \beta_1 \) and \( \beta_2 \) from Equation (16). The correlations documented in Figure 2 survive after controlling for this richer set of covariates and are all statistically significant at the 1% level. Relative to borrowers with low-DTI loans, the credit score of borrowers with high-DTI loans increases by 10.7 after the DTI limit is introduced. The interest rate they receive on their loans correspondingly declines by 5.5 basis points.

Additionally, I estimate the difference-in-differences specification in Equation (16) using the LTV ratio as the dependent variable. The coefficient on the interaction term is negative and statistically significant at the 1% level. Table 9 contains estimated values for \( \beta_1 \) and \( \beta_2 \) and shows that, relative to low-DTI loans, the LTV ratio of high-DTI loans declined by 5.5 percentage points after the DTI limit came into effect.

\(^{58}\) \( X_i \) includes dummy variables for the state in which the property is located, loan purpose (i.e., purchase or refinance), type of property, number of units on the property, and whether the borrower is a first-time homebuyer. Including \( Policy_t \) as a separate regressor is unnecessary because it is absorbed by the quarter dummy variables.

\(^{59}\) The results are robust to estimating the regression for the full sample of loans.
### Table 8. Effect of the 45% DTI limit on default risk and interest rates at origination

<table>
<thead>
<tr>
<th></th>
<th>(1) Credit score</th>
<th>(2) Interest rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI &gt; 45%</td>
<td>-1.270*** (0.000)</td>
<td>0.002** (0.023)</td>
</tr>
<tr>
<td>DTI &gt; 45% × Policy</td>
<td>10.705*** (0.000)</td>
<td>-0.055*** (0.000)</td>
</tr>
</tbody>
</table>

| N                   | 3,061,386        | 3,061,386             |
| R²                  | 0.139            | 0.909                 |

*p-level in parentheses. Robust standard errors are clustered at the state level.
* p < 0.10, ** p < 0.05, *** p < 0.01.

**Notes:** The two rows contain estimates for the coefficients $\beta_1$ and $\beta_2$ from the specification in Equation (16). Source: Freddie Mac Single Family Loan-Level Dataset.

### Table 9. Effect of the 45% DTI limit on LTV ratios at origination

<table>
<thead>
<tr>
<th></th>
<th>(1) LTV ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI &gt; 45%</td>
<td>-0.073 (0.105)</td>
</tr>
<tr>
<td>DTI &gt; 45% × Policy</td>
<td>-5.363*** (0.000)</td>
</tr>
</tbody>
</table>

| N                   | 3,061,386        |
| R²                  | 0.238             |

*p-level in parentheses. Robust standard errors are clustered at the state level.
* p < 0.10, ** p < 0.05, *** p < 0.01.

**Notes:** The two rows show estimates for the coefficients $\beta_1$ and $\beta_2$ from the specification in Equation (16). Source: Freddie Mac Single Family Loan-Level Dataset.
To ensure that the results are not driven by the fact that the DTI ratio on a given loan is large but, rather, a change exactly at the 45% limit, I follow DeFusco, Johnson and Mondragon (2020) in estimating a flexible difference-in-differences specification that allows the effect of the policy to vary with the DTI ratio. The regression equation is

$$y_{it} = \alpha + \sum_{k=40}^{50} \left[ \beta_{1k} \mathbb{1}_{DTI_i = k} + \beta_{2k} (\mathbb{1}_{DTI_i = k} \times Policy_t) \right] + \gamma_t + X_i' \delta + \varepsilon_{it},$$  \hspace{1cm} (17)

where $\mathbb{1}_{DTI_i = k}$ is an indicator variable that takes a value of 1 if the DTI ratio (in percent) of loan $i$ at origination is equal to $k \in \{41, 42, \ldots, 50\}$ and all other terms are as previously defined in Equation (16). I make $k = 45$ the omitted category such that $\beta_{2k}$ estimates the differential change in $y_{it}$ for loans originated with a DTI ratio equal to $k$ relative to loans with a DTI ratio of 45% after the policy is introduced. I estimate Equation (17) for a sub-sample of loans with DTI ratios between 40% and 50%, and robust standard errors are clustered at the state level.

Figure 12 plots point estimates for $\{\beta_{2k}\}_{k=40}^{50}$ and their respective 95% confidence intervals when Equation (17) is estimated with the credit score as the dependent variable. Relative to borrowers with DTI ratios equal to 45%, the credit score of borrowers who receive high-DTI mortgages increases by 14 after the DTI limit is introduced. Figure 13 plots point estimates for coefficients on the interaction terms and their respective 95% confidence intervals using the interest rate as the dependent variable. Relative to borrowers with DTI ratios equal to 45%, borrowers with high-DTI mortgages receive interest rates that are on average 7 basis points higher after the DTI limit is introduced.

Note that the differential changes in default risk estimated using the flexible difference-in-differences specification are larger than estimated using the baseline specification. This is because the control group in the baseline specification contains all loans with DTI ratios between 40% and 45%, whereas, in this instance, the control group only consists of borrowers exactly on the limit. As already shown in Figure 2, these borrowers have the highest default risk in the sample.
Figure 12. Effect of the 45% DTI limit on credit scores at origination

Notes: Blue squares correspond to estimates of $\beta_k$ for $k \in \{40, 41, \ldots, 50\}$ from the specification in Equation (17), where $k = 45$ is the omitted category. Blue bars denote the 95% confidence interval around the point estimates. Source: Freddie Mac Single Family Loan-Level Dataset.

Figure 13. Effect of the 45% DTI limit on interest rates at origination

Notes: Blue squares correspond to estimates of $\beta_k$ for $k \in \{40, 41, \ldots, 50\}$ from the specification in Equation (17), where $k = 45$ is the omitted category. Blue bars denote the 95% confidence interval around the point estimates. Source: Freddie Mac Single Family Loan-Level Dataset.
A.3 Mortgage maturity in the data

The baseline empirical analysis focuses on mortgages with 30-year terms because this is the most common residential mortgage contract in the United States. However, the debt payment-to-income ratio on a mortgage in part depends on its maturity. All else equal, increasing the term of the loan mechanically decreases the size of the debt payment and by extension the DTI ratio of the borrower. To the extent that the baseline analysis excludes loans with shorter maturities and, potentially, borrowers with higher DTI ratios, this may lead the empirical findings to excludes the types of households who may be most affected by the introduction of the 45% DTI limit.

Table 10 displays summary statistics for the sample of loans chosen according to the criteria described in Section 3.1 but without the 30-year maturity restriction. The mean term of loans in this broader sample is 26.3 years, and mortgages with 30-year terms account for 74% of them. Of the remaining 26% of loans, the vast majority have shorter terms. Interestingly, mortgages with shorter terms have somewhat lower DTI ratios, not higher ones. Consequently, between 2005 and 2016, 9% of such loans are high-DTI loans, compared to 15% of mortgages with 30-year terms. Mortgages with shorter maturities are also associated with higher credit scores, smaller balances, and lower interest rates. It is likely that a borrower composition effect can account for the observed positive relationship between loan maturity and DTI ratios in the dataset. Altogether, the summary statistics suggest that the types of households who choose mortgages with shorter terms in equilibrium are unlikely those for whom the 45% DTI limit is likely to bind.

To check that my empirical findings are robust to the focus on 30-year mortgages, I estimate the difference-in-differences specification in Equation (16) for this broader sample of loans. As before, I estimate this regression for loans in the 5-percentage point window around the 45% DTI cutoff and use the credit score and interest rate as dependent variables in turn. Estimated values for $\beta_1$ and $\beta_2$ are in Table 11, where $\beta_2$ is the coefficient of interest and captures the differential change in the dependent variable for high-DTI loans relative to low-DTI loans after the introduction of the 45% DTI limit in 2009Q1. The results documented in the main body of the text are largely unchanged when estimated for this sample of loans with varying maturity. Compared to low-DTI loans, the credit score on those with DTI ratios above 45% increases by 9.9 and the interest rate decreases by 6.7 basis points. I also verify that the LTV ratio on high-DTI loans decreases by more relative to low-DTI loans following the policy change for mortgages of varying maturities.
Table 10. Loan-level summary statistics for varying loan maturities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Term &lt; 30</th>
<th>Term = 30</th>
<th>Term &gt; 30</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI ratio (%)</td>
<td>30.62</td>
<td>34.64</td>
<td>42.37</td>
<td>33.57</td>
</tr>
<tr>
<td>Credit score</td>
<td>756.27</td>
<td>744.39</td>
<td>708.78</td>
<td>747.53</td>
</tr>
<tr>
<td>Interest rate (%)</td>
<td>4.15</td>
<td>5.01</td>
<td>6.55</td>
<td>4.78</td>
</tr>
<tr>
<td>LTV ratio (%)</td>
<td>62.13</td>
<td>72.81</td>
<td>74.35</td>
<td>69.97</td>
</tr>
<tr>
<td>Loan amount (000s)</td>
<td>179.12</td>
<td>224.16</td>
<td>224.83</td>
<td>212.18</td>
</tr>
<tr>
<td>High-DTI share</td>
<td>0.09</td>
<td>0.15</td>
<td>0.40</td>
<td>0.13</td>
</tr>
<tr>
<td>Refinance share</td>
<td>0.87</td>
<td>0.56</td>
<td>0.66</td>
<td>0.64</td>
</tr>
<tr>
<td>First-time homebuyer share</td>
<td>0.04</td>
<td>0.16</td>
<td>0.13</td>
<td>0.12</td>
</tr>
</tbody>
</table>

N: 3,783,071 10,440,989 5,242 14,229,302

Notes: The DTI ratio, credit score, LTV ratio, and loan amount at origination are reported as whole numbers in the dataset. Source: Freddie Mac Single Family Loan-Level Dataset.

Table 11. Effect of the 45% DTI limit on loan originations: all maturities

<table>
<thead>
<tr>
<th></th>
<th>(1) Credit score</th>
<th>(2) Interest rate (%)</th>
<th>(3) LTV ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI &gt; 45%</td>
<td>−1.258***</td>
<td>0.002*</td>
<td>−0.081*</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.076)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>DTI &gt; 45% × Policy</td>
<td>9.889***</td>
<td>−0.067***</td>
<td>−4.865***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

N: 3,849,413 3,849,413 3,849,413

R²: 0.138 0.875 0.227

Notes: The two rows contain estimates for the coefficients β₁ and β₂ from the specification in Equation (16) for a broader sample of loans with varying maturity. Source: Freddie Mac Single Family Loan-Level Dataset.
B Model appendix

B.1 Equilibrium definition

I solve for the stationary recursive equilibrium of the model. To establish notation, I define the state space, holding age \( j \) fixed, as \( W \equiv A \times H \times D \times P \times M \times Q \times R_M \times Z \). Let the \( \sigma \)-algebra \( \Sigma_W \) be defined as \( B_A \otimes B_H \otimes B_M \otimes B_Q \otimes B_{R_M} \otimes P(D) \otimes P(P) \otimes P(Z) \), where \( B_A, B_H, B_M, B_Q, \) and \( B_{R_M} \) are the Borel \( \sigma \)-algebras on \( A, H, M, Q, \) and \( R_M \), respectively, and \( P(D), P(P), \) and \( P(Z) \) are the power sets of \( D, P, \) and \( Z \), respectively. Define \( \Omega \equiv A \times H \times D \times P \times M \times Q \times R_M \times Z \) as the typical subset of \( \Sigma_W \).

For a given parameterization of the model and a measure of age-1 households \( \mu_1(\omega) \), a stationary recursive equilibrium consists of

1. household value functions \( \{ V^R_j(\omega), V^M_j(\omega), V^P_j(\omega), V^D_j(\omega) \} \) and policy functions
   \( \{ c_j(\omega), s_j(\omega), a'_j(\omega), h'_j(\omega), m'_j(\omega), q'_j(\omega) \} \);
2. a mortgage interest rate schedule \( r'_{m,j}(\omega) \); and
3. a stationary measure \( \Lambda^*_j(\Omega) \);

such that,

1. given \( r'_{m,j}(\omega) \), household value and policy functions solve the optimization problems
   in Equations (7), (8), (9), (10), (12), and (13);
2. given the household’s value and policy functions, \( r'_{m,j}(\omega) \) is such that financial
   intermediaries’ zero-expected profit condition in Equation (15) is satisfied on a loan-
   by-loan basis; and
3. the invariant probability measure satisfies
   \[
   \Lambda^*_{j+1}(\Omega) = \int_{\Omega} Q_j(\omega, \Omega) \left[ \Lambda^*_j(d\omega) + \mu_1(d\omega) \right]
   \]
   for all \( \Omega \in \Sigma_W \), where \( Q_j(\omega, \Omega) \) is the conditional probability that an age-\( j \) household
   in state \( \omega \) transitions to the set \( \Omega \) at age \( j + 1 \) and defined as

   \[
   Q_j(\omega, \Omega) \equiv 1_{a'_j(\omega) \in A, h'_j(\omega) \in H, m'_j(\omega) \in M, q'_j(\omega) \in Q, r'_{m,j}(\omega) \in R, a_j(\omega) \in A, h_j(\omega) \in H, m_j(\omega) \in M, q_j(\omega) \in Q} \sum_{\delta' \in D} \sum_{p' \in P} \sum_{z' \in Z} \pi(\delta') \pi(p'|p) \pi(z'|z).
   \]
B.2 Parameterizing the income process

To parameterize the deterministic age-dependent component of income, I follow Kaplan and Violante (2014) in regressing a quartic polynomial in age on log annual household income of households whose heads are between ages 25–65 from the 1999–2017 waves of the Panel Study of Income Dynamics (PSID). Using the estimated regression coefficients, I compute fitted values for log household income. These fitted values are the sequence \( \{\chi_j\}_{j=1}^{T_R-1} \) in the model, normalized so that log income of age-1 household is 0.

To parameterize the function for pension income \( \Phi (y_{T_R-1}(z)) \), I follow the procedure described by Guvenen and Smith (2014). For a given \((\rho_z, \sigma_z)\) pair, I simulate earnings for a panel of 100,000 households of working age and regress their average labor earnings on earnings at age \( T_R - 1 \). I use the estimated regression coefficients to predict average lifetime earnings \( \log \hat{y} \) for each possible realization of income at age \( T_R - 1 \), i.e., \( \log y_{T_R-1}(z) = \chi_{T_R-1} + z \). Let \( \log \bar{y} \) be the economy-wide average annual labor earnings and define \( \log \tilde{y} \equiv \log \hat{y} / \log \bar{y} \). The function for pension income estimated by Guvenen and Smith (2014) is

\[
\Phi (y_{T_R-1}(z)) = \begin{cases} 
\log \tilde{y} [0.9 \log \tilde{y}] & \text{if } \log \tilde{y} \leq 0.3 \\
\log \tilde{y} [0.27 + 0.32 (\log \tilde{y} - 0.3)] & \text{if } 0.3 < \log \tilde{y} \leq 2 \\
\log \tilde{y} [0.81 + 0.15 (\log \tilde{y} - 2)] & \text{if } 2 < \log \tilde{y} \leq 4.1 \\
1.13 \log \tilde{y} & \text{if } \log \tilde{y} > 4.1.
\end{cases}
\]

Figure 14 plots the median life-cycle income profile that results from these procedures.

B.3 Estimating the DTI offset

According to the Freddie Mac Single-Family Seller/Servicer Guide, liabilities that must be considered when computing a borrower’s debt payment-to-income ratio are their monthly housing expense, payments on all installment debts (e.g., student loans), payments on revolving accounts (e.g., credit cards), child support, and alimony. Monthly housing expense consists of principal and interest payments on the mortgage, property hazard insurance premiums, real estate taxes, homeowners association (HOA) dues, and other expenditures where applicable.\(^{60}\)

I directly calibrate the DTI offset from micro data. The offset is the sum of (1) non-first mortgage debt service payments and (2) other housing expenses, both scaled relative to income. I use the 2016 Survey of Consumer Finances (SCF) to parameterize the first

\(^{60}\)See https://guide.freddiemac.com/app/guide/content/a_id/1000663.
component of the DTI offset. A household’s non-mortgage DTI ratio is the sum of their non-mortgage revolving and non-revolving ratios; DTI ratios on second and third mortgages; and the ratio of alimony payments to income. To control for the effect that household observables may have on the non-mortgage DTI ratio, I estimate the regression

$$\varsigma_i = \alpha + \sum_{j=1}^{4} \beta_j \text{age}_i^j + \gamma \left( \frac{w_i}{y_i} \right) + \delta \left( \frac{a_i}{y_i} \right) + \zeta_1 \mathbb{1}_{h_i>0} + X_i'\eta + \varepsilon_i,$$

where \(\varsigma_i\) is the non-mortgage DTI ratio of household \(i\), \(\alpha\) is a constant term, \(\{\text{age}_i^j\}_{j=1}^{4}\) is a quartic polynomial in age, \(w_i/y_i\) is the net worth to income ratio, \(a_i/y_i\) is the liquid wealth to income ratio, \(\mathbb{1}_{h_i>0}\) is an indicator variable for homeownership, \(X_i\) is a vector of household demographic characteristics, and \(\varepsilon_i\) is an error term.\(^{61}\) I include these three balance sheet variables in the regression because they have direct model equivalents. I report estimated regression coefficients in Table 12.

\(^{61}\)\(X_i\) includes indicator variables for the head of household’s race, education, sex, marital status, and labor force participation, as well as the number of children in the household.
Table 12. Estimated coefficients from the DTI offset regression equation

<table>
<thead>
<tr>
<th></th>
<th>(1) Non-mortgage DTI ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>age</td>
<td>-0.024*</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
</tr>
<tr>
<td>age^2</td>
<td>0.001**</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
</tr>
<tr>
<td>age^3</td>
<td>-0.000**</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
</tr>
<tr>
<td>age^4</td>
<td>0.000**</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>w/y</td>
<td>0.000***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
</tr>
<tr>
<td>a/y</td>
<td>-0.001***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>(1_{h&gt;0})</td>
<td>-0.011***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.410***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>16,419</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.035</td>
</tr>
</tbody>
</table>

p-level in parentheses, * p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The rows report estimated values of \(\{\beta_j\}_{j=1}^4\), \(\gamma\), \(\delta\), \(\zeta\), and \(\alpha\) from the DTI offset regression in Equation (20). Source: 2016 SCF.
To parameterize the second component of the DTI offset, I use the 2017 American Housing Survey (AHS). Housing expenses relative to income are the sum of monthly property taxes (PROTAXAMT), home insurance premiums (INSURAMT), HOA fees (HOAAMT), and lot rent (LOTAMT), all scaled by monthly household income (HINCP). For simplicity, I assume that all agents in the model have the same housing expense ratio and set it equal to the average in the data, which is 8%.

When solving the problem of a household who chooses to own and obtain a new mortgage loan in Equation (10), I use the estimated values of \( \alpha, \{ \beta_j \}_{j=1}^4, \gamma, \delta, \) and \( \zeta \) to predict the non-mortgage DTI ratio given the household’s age and state. I add to that the housing expense ratio calculated from the AHS. In Figure 15, I show that this procedure yields a DTI offset that, in the stationary distribution of borrowers in the model, is monotonically decreasing in age. On average, the offset across households obtaining new loans is 20.1%.

### B.4 Calculating calibration targets from the Survey of Consumer Finances

In order to reflect the most recent financial conditions of U.S. households, I use the 2016 SCF summary extract file to compute cross-sectional and life-cycle moments of household balance sheets targeted in the calibration. Net worth is the sum of liquid assets and home equity. Following Kaplan and Violante (2014), liquid assets are the sum of assets held in checking accounts, savings accounts, call accounts, directly held mutual accounts, directly held bonds, and directly held stocks. Home equity is the difference between the value of
primary residential real estate and debt outstanding on the first mortgage secured by the primary residence.\footnote{Because relatively few households in the SCF report having more than one loan secured by their primary residence or owning a second home, the inclusion of second mortgages (e.g., home equity loans or home equity lines of credit) is quantitatively unimportant for my results.} I define household income as the sum of wage and salary income; income from unemployment insurance and benefits; and Social Security and pension income. I limit my sample to households whose heads are between ages 25–80, have strictly positive household income, and below the top 1% of the net worth distribution. All moments are calculated using the SCF sample weights.

I use the bottom 99% of the net worth distribution to calculate calibration targets because the SCF over-samples households who are likely to be relatively wealthy in order to increase representation of the upper tail of the wealth distribution and to make possible analyses of less widely held asset classes. These families correspond to the “list sample” because they are selected using specially edited individual tax returns provided by the Internal Revenue Service. In the 2004 SCF, for example, the list sample accounts for only 15% of observations in the bottom 95% of the wealth distribution but 88% of observations in the top 5% percent.\footnote{See https://www.federalreserve.gov/econresdata/scf/files/scf2001list.sampleredesign9.pdf.}

### B.5 Additional results on borrower selection into high-DTI mortgages

Figure 16 compares the \textit{ex ante} default probability of borrowers as a function of their DTI ratio in the model to their nearest empirical counterpart, the \textit{ex post} probability that a borrower is delinquent on their mortgage one year after payments begin. I use this comparison because the model does not contain a formal notion of a credit score. A credit score in reality is a backward-looking proxy of a borrower’s ability to repay, whereas the default probability in the structural model is a forward-looking measure. To compute the \textit{ex post} delinquency probability, I merge the monthly performance files of the Freddie Mac Single Family Loan-Level Dataset to the quarterly origination files. From the monthly performance files, I construct an indicator variable equals 1 if a borrower is at least 30 days delinquent on their loan one year after their first payment is due. In reality, borrowers are often delinquent on their payments for some time before a formal foreclosure process is initiated. In the model, delinquency and default are identical decisions: a debtor who does not make a mortgage payment necessarily loses their house as well. Because the definitions of default in the model and the data differ, one should not expect the quantitative magnitudes to line up exactly. Nevertheless, the model generates predictions that are qualitatively similar to what is found in the data.\footnote{This result is robust to using more or less more or less strict definitions of default—e.g., delinquent at any point in the life of the loan, more than 360 days delinquent, property has been repossessed by the lender—to construct the \textit{ex post} probability of default in the data.}
Figure 16. Default probabilities as a function of DTI ratio at origination in the model vs. data

Notes: “With DTI limit” refers to the baseline calibration of the model. “Without DTI limit” refers to an alternative calibration in which the default risk threshold for a high-DTI loan is fully relaxed. Default probability in the model is the \textit{ex ante} probability that an age-$j$ household in state $\omega$ optimally defaults at age $j+1$. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.

Figure 17. LTV ratio as a function of the DTI ratio at origination in the model vs. data

Notes: “With DTI limit” refers to the baseline calibration of the model. “Without DTI limit” refers to an alternative calibration in which the default risk threshold for a high-DTI loan is fully relaxed. “LTV ratio” refers to LTV ratio at origination. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.
In Figure 17, I compare LTV ratios at origination in the model and the data for the pre- and post-DTI limit periods. In line with the data, the model predicts that, with the current DTI limit in place, the LTV ratios of loans above the limit are smaller than those of loans just on the limit. Without the DTI limit, LTV ratios remain elevated past the 45% cutoff. There is much greater dispersion in LTV ratios at origination in the model than in the data. This likely reflects the fact that mortgages are the only form of debt available to households in the model. In reality, many households have access to unsecured credit in order to insure against smaller negative shocks to cash on hand. As a result, the model over-predicts the share of households who obtain mortgages with small balances at origination: the mean LTV ratio is 63% in the model, compared to 76% in the data.\(^\text{65}\) This quantitative feature of the model could be addressed by relaxing the no-borrowing constraint on the liquid asset to allow households some ability to smooth consumption through non-mortgage debt, then calibrating the borrowing limit on liquid wealth to match the observed share of households who carry positive unsecured debt.

**B.6 Additional results on the heterogeneity of default risk at the DTI limit**

In Figure 18, I show that borrowers who choose new loans with DTI ratios between 43% and 45% at origination in the stationary distribution of the model are non-uniform in their underlying characteristics and, thus, their default risk. To do so, I separate these borrowers into two groups based on whether their likelihood of default is less than or greater than the qualifying default risk threshold for a high-DTI loans (“safe” and “risky,” respectively). I also include high-DTI borrowers for comparison. This figure shows that “safe bunched” borrowers look more like their counterparts above the DTI limit. “Risky bunched” borrowers share more characteristics with households who are constrained along the extensive margin by the DTI limit. These differences in household characteristics are also reflected in the loan size and interest rate received by the different subgroups. These results demonstrate that the model qualitatively matches the fact that borrowers bunched below the 45% DTI cutoff exhibit greater variance in their default risk than households located above the cutoff.

\(^{65}\)The distribution of LTV ratios at origination in the model does exhibit a mass point at the statutory maximum of 85%, similar to the spikes at institutional LTV limits documented by Greenwald (2018).
**Figure 18.** High-DTI and bunched borrowers in the model

Notes: “Safe (risky) bunched” refers to borrowers who optimally choose a low-DTI loan with a DTI ratio at origination between 43% and 45% and have an endogenous default probability less (greater than) than $\Psi = 0.025\%$. “High DTI” refers to borrowers who optimally choose a high-DTI loan. Home equity and net worth are scaled by income. Loan size is in units of the numeraire. A constrained household is an agent who, in the absence of the default risk threshold, would have optimally chosen to obtain a high-DTI loan. By construction, the share of constrained households among borrowers choosing high-DTI loans is 0.
B.7 Calibrating the cost of a non-qualified mortgage under Dodd-Frank

The Dodd-Frank Act of 2010 delegated the implementation of the ability-to-repay rule to the newly created Consumer Financial Production Bureau (CFPB). In January 2013, the Consumer Financial Protection Bureau (2013) announced the final rule and officially entered it into the Federal Register. I follow a cost-benefit analysis from the final rule to calibrate the value of $\gamma_H$, the cost of default on a non-qualified mortgage to lenders.\footnote{The final rule can be found at \url{https://www.federalregister.gov/documents/2013/01/30/2013-00736/ability-to-repay-and-qualified-mortgage-standards-under-the-truth-in-lending-act-regulation-z}.}

If a borrower who defaults on a non-qualified mortgage brings a successful legal claim under the ability-to-repay rule, the lender who originated that mortgage is liable for up to three years of fees and finance charges; the borrower’s legal expenses; and statutory damages under the Truth in Lending Act (TILA).\footnote{The Truth in Lending Act of 1968 requires that lenders disclose credit terms to consumers in a “meaningful way.” Failure to do so can result in the lender being liable for statutory damages. The ability-to-repay rule is implemented through TILA, and the Dodd-Frank Act authorized the transfer of TILA’s rule-making authority from the Federal Reserve to the CFPB. See \url{https://www.consumerfinance.gov/policy-compliance/rulemaking/regulations/1026/}.} Borrowers who default on a qualified do not have standing under this rule to sue the lender; thus, I leave the calibrated value of $\gamma_L$ unchanged. To compute the fees and finance charges owed by the lender, I calculate the mean of three years of interest payments on mortgages originated to borrowers with an above-average probability of default in the stationary distribution of the model. This amounts to 0.48 units of the numeraire consumption good.\footnote{Among borrowers with an above-average probability of default, the mean loan balance at origination is 5.82 and the mean interest rate is 2.76%.} I then add to that the mortgage origination cost $\kappa_m$. The CFPB estimates that combined legal expenses of a lender and borrower are $34,500 and additionally assumes that a borrower is rewarded $4,000 in statutory damages under TILA. Together, the legal costs equal 74\% of mean U.S. household income, equivalent to 1.03 units of the numeraire.\footnote{Mean household labor income in the sub-sample of households in the 2016 SCF to which I calibrate the model is $52,108 in 2013 CPI-U-RS adjusted dollars. Mean household labor income in the model is 2.23 units of the numeraire.} Finally, I add to this the housing transaction cost $\kappa_h$ to account for costs generated by the foreclosure itself. In total, the resource loss suffered by a lender due to default on a non-qualified mortgage is 2.374. This is about 4 times greater than the cost of default on a qualified mortgage to lenders.

In reality, not every borrower who is unable to repay their mortgage will bring a case against the responsible lender. That decision depends on, among other factors, whether the borrower lives in a judicial or non-judicial foreclosure state and their willingness and/or ability to obtain legal representation. Nonetheless, substantial evidence suggests that...
mortgage lenders are worried about these regulations. Fuster, Lo and Willen (2017) estimate that, between 2008 and 2014, the price of intermediation in the mortgage market increased by around 30 basis points per year and that this trend appears to be driven by increased net costs of mortgage servicing and heightened aversion to liability risk among lenders. Kim et al. (2018) document legal actions the GSEs and the U.S. federal government took after the Great Recession in response to improper loan originations. In light of this, the fact that Dodd-Frank creates the potential for increased legal claims against mortgage lenders—even if there is uncertainty about how many will claims will ultimately be brought—should be taken seriously.

I show in Table 13 that results from my policy counterfactual that compares the original DTI-based ability-to-repay rule to the revised price-based rule are robust to different values for $\gamma_H$. The aggregate welfare gain remains positive throughout but decreases as the cost of foreclosure on a non-qualified mortgage to lenders rises. Aggregate homeownership and default rates are also decreasing in $\gamma_H$.

**B.8 Defining consumption-equivalent welfare change**

Following Gete and Zecchetto (2018), I define an age-$j$ household’s composite consumption as

$$C_j \equiv \left( \alpha_j c_j^{1-\vartheta} + (1-\alpha_j) s_j^{1-\vartheta} \right)^{1\over 1-\vartheta}.$$ 

Consumption-equivalent welfare change $\Delta C_j(\omega)$ is the percent change in the composite consumption of age-$j$ household in state $\omega$ needed to make them exactly indifferent between the stationary economies under the two versions of the ability-to-repay rule. The assumption

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---

The CFPB, for one, concedes that its estimate of litigation costs relies on “very conservative (likely unrealistic) assumptions.” There is at least one known instance of a household bringing a successful suit under the ability-to-repay rule, G. Elliot v. First Federal Community Bank: [https://law.justia.com/cases/federal/appellate-courts/ca6/19-3690/19-3690-2020-07-08.html](https://law.justia.com/cases/federal/appellate-courts/ca6/19-3690/19-3690-2020-07-08.html).
of CES preferences yields the closed-form solution

\[ \Delta C_j(\omega) = \left( \left( \frac{\tilde{V}_j(\omega)}{V_j(\omega)} \right)^{\frac{1}{1-\sigma}} - 1 \right) \times 100, \]

where \( V_j(\omega) \) and \( \tilde{V}_j(\omega) \) are the value functions of an age-\( j \) household in state \( \omega \) under the current and new policies, respectively. The aggregate welfare change in consumption-equivalent terms is therefore

\[ \Delta C = \int \Delta C_j(\omega) \Lambda_j(d\omega), \]

where \( \Lambda_j(\omega) \) is the stationary distribution of households over states under the initial policy.

C  Numerical solution of the model

C.1  Simplifying the state space

Following Berger and Vavra (2015) and Kaplan and Violante (2014), I redefine the state space of the model by noting that, conditional on the household adjusting their house size, it is sufficient to track their home equity. In other words, the household only cares about the net cash on hand that results from selling their existing house and repaying their outstanding mortgage debt, not its composition. I define the home equity \( e \) of a household with house size \( h \), house price \( p \), depreciation shock \( \delta \), mortgage debt \( m \), and interest rate \( r_m \) as

\[ e(h, \delta, p, m, r_m) \equiv (1 - \delta)ph - (1 + r_m)m. \]  (21)

The cash-on-hand formulation significantly reduces the dimensionality of the state vector of a household who chooses to rent or chooses to obtain a new mortgage. Because the optimization problem associated with obtaining a new loan is the most computationally intensive part of solving the household’s problem, this dramatically reduces overall computation time. Let \( \omega_A \equiv (a, h, e, p, z) \) be the current state of an age-\( j \) household solving either of those optimization problems.\(^{71}\)

A household who continues with an existing mortgage does not adjust their house size, so I continue to track their outstanding debt, mortgage interest rate, and contract type separately. As a result, I define their optimization problem in terms of the original vector of

\(^{71}\)Tracking \( h \) as a separate state variable is needed to determine if a household pays the housing adjustment cost. Tracking the depreciation shock \( \delta \) is not, both because it is subsumed in the definition of home equity in Equation (21) and i.i.d. by assumption.
state variables from Section 4.2. For notational consistency, I relabel the state vector of an age-\(j\) household who continues with an existing loan as \(\omega_N \equiv (a, h, \delta, p, m, q, r_m, z)\).

Finally, note that, conditional on defaulting, home equity is irrelevant to the household’s problem because default, by definition, sets both their housing stock and outstanding debt equal to zero. The mortgage contract type remains a state variable because the household’s decision to default affects the expected discounted cash flow of a loan. Thus, I define the state vector of an age-\(j\) household who chooses to default as \(\omega_D \equiv (a, p, q, z)\).

### C.2 Redefining optimization problems

I rewrite the household’s optimization problem for the redefined state space. The expected discounted lifetime utility of an age-\(j\) household who rents in state \(\omega_A\) is

\[
W^R_j(\omega_A) = \max_{x_j(\omega_A)} \{V^R_j(\omega_A), V^M_j(\omega_A)\}.
\]

The expected discounted lifetime utility of an age-\(j\) household who owns in state \(\omega_N\) is

\[
W^O_j(\omega_N) = \max_{x_j(\omega_N)} \{V^R_j(\omega_A), V^M_j(\omega_A), V^P_j(\omega_N), V^D_j(\omega_D)\},
\]

where \(\omega_A = (a, h, e(h, \delta, p, m, r_m), p, z)\).

If renting, a household solves

\[
V^R_j(\omega_A) = \max_{c, s, a'} u_j(c, s) + \beta \mathbb{E}_{p', z'} | p, z W^R_{j+1}(\omega'_A)
\]

s.t.

\[
c + Rps + a' \geq y_j(z) + (1 + r) a + e - 1_{h \neq 0} \kappa_h
\]

\[
a' \geq 0
\]

\[
\omega'_A = (a', 0, 0, p', z').
\]

Note that, if a household begins the next period as a renter, taking expectations over the depreciation shock is unnecessary because, by definition, they will have a house size of 0.
If obtaining a new loan, a household solves

\[
V_j^M (\omega_A) = \max_{c,a',h',m',q' \in \{L,H\}} u_j (c, h') + \beta \mathbb{E} \delta',p',z'|p,z \mathbb{W}_{j+1}^{O} (\omega'_{N})
\]

s.t.

\[
c + a' + 1_{h' \neq h} (ph' - \kappa_h) \leq y_j (z) + (1+r) a + e - (1 - 1_{h' \neq h}) ph + m' - 1_{m' > 0} \kappa_m
\]

\[
m' \leq \theta ph'
\]

\[
\pi_{min,j} (m',r'_{m,j} (\omega_A)) \leq \lambda_j (\omega_A) y_j (z)
\]

\[
\lambda_j (\omega_A) = \begin{cases} 
\lambda - \varsigma_j (\omega_A) & \text{if } \psi_j (\omega_A) > \Psi \\
\infty & \text{if } \psi_j (\omega_A) \leq \Psi
\end{cases}
\]

\[
q'_j (\omega_A) = \begin{cases} 
L & \text{if } \pi_{min,j} (m',r'_{m,j} (\omega_A)) \leq (\lambda - \varsigma_j (\omega_A)) y_j (z) \\
H & \text{if } \pi_{min,j} (m',r'_{m,j} (\omega_A)) > (\lambda - \varsigma_j (\omega_A)) y_j (z)
\end{cases}
\]

\[
a' \geq 0
\]

\[
\omega'_N = (a',h',\delta',p',m',q',r'_{m,j} (\omega_A),z')
\]

where the expected probability of default is computed as

\[
\psi_j (\omega_A) = \mathbb{E} \delta',p',z'|p,z \{1_{x_{j+1} (\omega_D) = D} | x_j (\omega_A) = M \}
\]

This formulation of the flow budget constraint allows me to compute the housing maintenance costs and loan repayment required by a mortgage refinance using this lower-dimensional state vector, despite the household not adjusting their housing stock. A mortgage refinance occurs when \(1_{h' \neq h} = 0\) and \(1_{m' > 0} = 1\). Using the definition of home equity in Equation (21), the household’s cash on hand is

\[
y_j (z) + (1+r) a - \delta ph - (1+r_m) m + m' - 1_{m' > 0} \kappa_m.
\]

Therefore, knowledge of \(h\), \(e\), and \(p\) is sufficient for backing out the housing maintenance costs \(\delta ph\) and loan repayment \((1+r_m) m\).
If making a payment on an existing loan, a household solves

$$V^P_j (\omega_N) = \max_{c,a',m'} u_j (c, h) + \beta \mathbb{E} u^P_{j+1} (\omega'_N)$$

s.t.

$$c + \delta ph + a' \leq y_j (z) + (1 + r) a - (1 + r_m) m + m'$$
$$m' \leq (1 + r_m) m - \pi_{min,j} (m, r_m)$$
$$a' \geq 0$$
$$\omega'_N = (a', h, \delta', p', m', q, r_m, z') .$$

If defaulting, a household solves

$$V^D_j (\omega_D) = \max_{c,s,a'} u_j (c, s) - \xi + \beta \mathbb{E} u^D_{j+1} [\varphi W^R_{j+1} (\omega'_A) + (1 - \varphi) V^R_{j+1} (\omega'_A)]$$

s.t.

$$c + Rps + a' \leq y_j (z) + (1 + r) a$$
$$a' \geq 0$$
$$\omega'_A = (a', 0, 0, p', z') .$$

The financial intermediary’s problem presented in Section 4.3 remains unchanged. For notational consistency, I rewrite the present value of an existing mortgage held by an age-$j$ household in state $\omega_N$ as

$$\Pi_j (\omega_N) = \begin{cases} 
(1 + r_m) m & \text{if repay} \\
(1 - \delta) ph - \gamma (q) & \text{if default} \\
(1 + r_m) m - m'_j (\omega_N) + \frac{1}{1 + r + \phi} \mathbb{E} u^D_{j+1} (\omega'_N) \Pi_{j+1} (\omega'_N) & \text{otherwise,}
\end{cases}$$

(29)

where $\omega'_N = (a'_j (\omega_N), h'_j (\omega_N), \delta', p', m'_j (\omega_N), q'_j (\omega_N), r'_{m,j} (\omega_N), z')$. I rewrite the zero-profit condition on a mortgage originated to an age-$j$ household in state $\omega_A$ as

$$m'_j (\omega_A) = \frac{1}{1 + r + \phi} \mathbb{E} u^D_{j+1} (\omega'_N) \Pi_{j+1} (\omega'_N) ,$$

(30)

where $\omega'_N = (a'_j (\omega_A), h'_j (\omega_A), \delta', p', m'_j (\omega_A), q'_j (\omega_A), r'_{m,j} (\omega_A), z')$.

C.3 Discretization

I use the Rouwenhorst method described by Kopecky and Suen (2010) to discretize the first order Markov processes for shocks to income $z$ and house prices $p$. This method
yields grids for each shock and a unique matrix of associated transition probabilities. The grid for house size $h$ is $\{0, h, \ldots, \bar{h}\}$, where, by definition, $h = 0$ for existing renters and $h \in \{h, \ldots, \bar{h}\}$ for existing owners. The points on the housing grid are set according to the procedure described in Section 5.2. The bounds on the grid for mortgage debt $m$ are $[0, \theta \bar{p} \bar{h}]$, where $\bar{p}$ is the maximum possible house price. The bounds on the grid for the mortgage interest rate $r_m$ are $[r + \phi, \bar{r}_m]$. I confirm ex post that, in stationary equilibrium, the upper bound $\bar{r}_m$ does not bind. The grid for the depreciation shock $\delta$ is $\{\delta, \bar{\delta}\}$. The grid for the mortgage contract $q$ is $\{L, H\}$. The grid for liquid assets $a$ features more points clustered near the borrowing constraint. The bounds on the liquid asset grid are $[0, \bar{a}]$. I set $\bar{a}$ to $(1 - \delta') \bar{p} h - (1 + \bar{r}_m)m$. I confirm ex post that the upper bound on liquid assets is not binding in the stationary distribution of the model. The bounds on the grid for home equity $e$ are $[e, \bar{e}]$, where $e = (1 - \delta') \bar{p} h - (1 + \bar{r}_m)m$ and $\bar{e} = (1 - \delta) \bar{p} h - (1 + \bar{r}_m)m$. The home equity grid contains a point at 0.

Choices for the three endogenous state variables $a$, $m$, and $r_m$—as well as values of $e$ where applicable—are permitted to lie off the grid. I use linear interpolation to evaluate the value and policy functions at off-grid points when solving the model. I constrain the choice of $h'$ to the house size grid in order to capture the indivisibility of housing as an asset. To construct the transition function for households over states $Q_j(\omega_N, \Omega_N)$ and compute the stationary distribution $\Lambda^*_j(\Omega_N)$, I interpolate the household’s policy and value functions over finer grids for the three continuous endogenous state variables.

To evaluate expectations inside of the Bellman equations, I pre-compute integrals using the technique described by Judd et al. (2017). After I finish solving an age-$j$ household’s problem for all $\omega_A$, $\omega_D$, and $\omega_N$, I define and compute the following for $j < T$:

$$EW^R_j(\omega_A) \equiv E_{p', z'}|p, z} \max \{V^R_j(a, h, e, p', z'), V^M_j(a, h, e, p', z')\}$$
$$EW^O_j(\omega_N) \equiv E_{\delta', p', z'}|p, z} \max \{V^R_j(a, h, \bar{e}', p', z'), V^M_j(a, h, \bar{e}', p', z'), V^P_j(a, h, \delta', p', m, q, r_m, z'), V^D_j(a, p', q, z')\}$$,

where $\bar{e}' \equiv (1 - \delta') p'h - (1 + r_m)m$. $EW^R_j(\omega_A)$ and $EW^O_j(\omega_N)$ represent the household’s continuation values conditional on holding the endogenous state variables fixed.
I then compute the value functions of an age-\(j\) household as

\[
V^R_j (\omega_A) = u(e^R_j (\omega_A), s^R_j (\omega_A)) + \beta EW^R_{j+1} (a^R_j (\omega_A), 0, 0, p, z)
\]

\[
V^M_j (\omega_A) = u(e^M_j (\omega_A), h^M_j (\omega_A)) + \beta EW^O_{j+1} (a^M_j (\omega_A), \delta, p, m^M_j (\omega_A), q^M_j (\omega_A), r^M_{m,j} (\omega_A), z)
\]

\[
V^P_j (\omega_N) = u(e^P_j (\omega_N), h) + \beta EW^O_{j+1} (a^P_j (\omega_N), h, \delta, p, m^P_j (\omega_N), q, r_m, z)
\]

\[
V^D_j (\omega_D) = u(e^D_j (\omega_D), s^D_j (\omega_D)) + \beta [\varphi EW^R_{j+1} (a^D_j (\omega_D), 0, 0, p, z) + (1 - \varphi) V^R_{j+1} (a^D_j (\omega_D), 0, 0, p, z)],
\]

where continuation values are obtained by interpolating \(EW^R_{j+1} (\omega_A)\) and \(EW^O_{j+1} (\omega_N)\) at values for the endogenous state variables in the next period. I also pre-compute integrals for the financial intermediary’s problem, defining

\[
E \Pi_j (\omega_N) \equiv E \delta', p', z' | p, z \Pi_j (a, h, \delta', p', m, q, r_m, z'),
\]

such that the zero-profit condition is computed as

\[
m^M_j (\omega_A) = \frac{1}{1 + r + \phi} E \Pi_{j+1} (a^M_j (\omega_A), h^M_j (\omega_A), \delta, p, m^M_j (\omega_A), q^M_j (\omega_A), r^M_{m,j} (\omega_A), z).
\]

C.4 Solution algorithm

1. Solve the problem of a household in the last period of life to obtain \(V^R_T (\omega_A)\), \(V^M_T (\omega_A)\), \(V^P_T (\omega_N)\), and \(V^D_T (\omega_D)\), along with all associated policy functions. By assumption, \(m^R_T (\omega_N) = 0\) and \(m^P_T (\omega_A) = 0\). Next, compute the present value of cash flows associated with a mortgage held by an age-\(T\) household in state \(\omega_N\). If the household repays their loan, then

\[
\Pi_T (\omega_N) = (1 + r_m) m.
\]

If the household defaults, then

\[
\Pi_T (\omega_N) = (1 - \delta) ph - \gamma (q).
\]

2. Use backward induction to solve for value functions in Equations (24), (25), (27), and (28) for ages \(j < T\).

(a) Solving the problem of a household who chooses to rent, \(V^R_j (\omega_A)\):

(i) This option is available to all households.
(ii) The assumption of CES preferences over nondurable consumption and housing services implies

\[ s = \left( \frac{1 - \alpha}{\alpha R} \right)^{\frac{1}{\vartheta}} c. \]  

(31)

Use this expression to substitute out \( s \) in the household’s problem, and use the budget constraint to substitute out \( c \) from the flow utility function.

(iii) Solve for \( V^R_j (\omega_A) \) and \( a'^R_j (\omega_A) \) using Brent’s method. Let \( c \) be the lowest possible value of nondurable consumption.\(^{72}\) The requirement that \( c \geq \bar{c} \) characterizes the set of feasible solutions,

\[ a' \leq a' \leq y_j (z) + (1 + r) a + e - \mathbbm{1}_{h \neq 0} h - \bar{c} \left[ 1 + \left( \frac{1 - \alpha}{\alpha R^{1-\vartheta}} \right)^{\frac{1}{\vartheta}} \right]^{-1}, \]

where the no-borrowing constraint on the liquid asset implies \( a' = 0 \).

(iv) Find \( c_j^R (\omega_A) \) from the flow budget constraint and use Equation (31) to find \( s_j^R (\omega_A) \).

(v) By definition, \( h_j^R (\omega_A) = 0 \) and \( m_j^R (\omega_A) = 0 \). Because the household is not a debtor, \( q_j^R (\omega_A) \) and \( r_{m,j}^R (\omega_A) \) can be set to any arbitrary value.

(b) Solving the problem of a household who owns and obtains a new loan, \( V_j^M (\omega_A) \):

(i) This option is available to all households. When solving this problem for age-\( T \) households, \( m_j^M (\omega_A) \) is constrained to be 0 and the financial intermediary’s problem is skipped. These households can still choose to adjust their housing stock.

(ii) Hold \( r_{m,j}^R (\omega_A) \) fixed. Loop through all feasible \( h' \).\(^{73}\) For each feasible \( h' \), use Nelder-Mead to solve for \( a_j^M (\omega_A), m_j^M (\omega_A), \) and \( V_j^M (\omega_A) \). Maximum feasible debt is determined by the LTV and DTI limits,

\[ \bar{m}' = \min \{ \bar{m}_{ltv}', \bar{m}_{dti}' \}, \]

where

\[ \bar{m}_{ltv}' \equiv \theta ph'. \]

\(^{72}\) In the computation, \( c \) is set to 0.001.

\(^{73}\) In this context, feasibility means that, for a given idiosyncratic state and \( r_{m,j}^R (\omega_A), h' \) is in the household’s budget set and \( c \geq \bar{c} \) assuming \( m_j^L (\omega_A) = \bar{m}', a_j^L (\omega_A) = a', \) and \( r_{m,j}^L (\omega_A) = \Sigma_m. \)
and
\[
\bar{m}_{dti}' \equiv (\lambda - \varsigma_j (\omega_A)) y_j (z) \left[ r'_{m,j} (\omega_A) \left( \frac{1 + r'_{m,j} (\omega_A)}{1 + r'_{m,j} (\omega_A)} \right)^{T-j} \right]^{-1}.
\]

If the DTI limit is fully relaxed, then
\[
\bar{m}' = \bar{m}_{ltv}'.
\]

For a given \( \bar{m}' \), the set of feasible solutions is characterized by
\[
a' \leq a' \leq y_j (z) + (1 + r) a + e - (1 - 1_{h' \neq h}) ph + \bar{m}' - \kappa_m
\]
\[
- 1_{h' \neq h} (ph' + \kappa_h) - \zeta
\]
and
\[
\max \{ \zeta + 1_{h' \neq h} (ph' + \kappa_h) + a' + \kappa_m + (1 - 1_{h' \neq h}) ph - y_j (z) - (1 + r) a - e, 0 \} \leq m' \leq \bar{m}'.
\]

Conditional on the DTI limit being fully relaxed, compute the expected probability of default \( \psi_j (\omega_A) \) using Equation (26). Use the budget constraint to find \( c^M_{j} (\omega_A) \). At the end of this step, select the value of \( h' \) (and policy functions implied by that choice) that yields the highest value for the household.

(iii) Given a solution to the household’s problem found in the previous step, compute the financial intermediary’s profit using Equation (30).

(iv) A bisection algorithm is used to find the break-even interest rate on a newly originated mortgage \( r'_{m,T-1} (\omega_A) \). This algorithm exploits the fact that the financial intermediary’s profit is increasing in \( r'_{m,j} (\omega_A) \), all else equal, and searches over the interval \([r_m, \bar{r}_m]\).

(A) The interest rate received by a household of age \( T-1 \) who obtains a new loan \( r'_{m,T-1} (\omega_A) \) is \( r + \phi \). This follows from the fact that, if an age-\( T \) household repays their outstanding mortgage debt, then the zero profit condition is
\[
\frac{1}{1 + r + \phi} \left( 1 + r'_{m,T-1} (\omega_A) \right) m'_{T-1} = m'_{T-1},
\]
implying \( r'_{m,T-1} (\omega_A) = r + \phi \). Note that, if an age-\( T \) household defaults,
then, in equilibrium, an intermediary will not sell that household a mortgage contract in the previous period.

(B) If the financial intermediary’s profit is negative when \( r'_{m,j}(\omega_A) = \bar{r}_m \), then the option to obtain a mortgage is not available to the household in equilibrium and the expected probability of default \( \psi_j(\omega) \) is set to 1. If the intermediary’s profit is positive when \( r'_{m,j}(\omega_A) = \underline{r}_m \), then the household borrows at the rate \( r + \phi \) and the expected probability of default \( \psi_j(\omega) \) is set to 0. If the financial intermediary’s profit is positive when \( r'_{m,j}(\omega_A) = \bar{r}_m \) and negative when \( r'_{m,j}(\omega_A) = \underline{r}_m \), then an interior solution exists and bisection is used to solve for the equilibrium interest rate that earns the lender zero profits on the loan in expectation.

(v) Execute steps (ii)–(iv) assuming the DTI limit does and does not apply in the household’s problem. For each case, compute the household’s expected probability of default using Equation (26). If the probability of default exceeds the default risk threshold conditional on the DTI requirement being relaxed, then the household’s solution to this problem necessarily satisfies the statutory limit. Otherwise, select the case—i.e., with or without the DTI limit—that yields the highest value. Determine the contract type \( q'_j(\omega_A) \) implied by the household’s optimal loan choice, if applicable.

(c) Solving the problem of an owner who continues with an existing loan, \( V^P_j(\omega_N) \):

(i) This option is only available to existing homeowners \( (h > 0) \). Note that this problem is also solved by homeowners who do not have any debt. In this case, \( m'^P_j(\omega_N) = 0 \) and the owner only needs to solve for \( a'^P_j(\omega_N) \).

(ii) Use the budget constraint to substitute \( c \) out of the flow utility function.

(iii) Solve for \( a'^P_j(\omega_N), m'^P_j(\omega_N), \) and \( V^P_j(\omega_N) \) using Nelder-Mead. From the law of motion for mortgage debt, we have

\[ \bar{m}' = (1 + r_m) m - \pi_{min,j}(m, r_m), \]

where \( \pi_{min,j}(m, r_m) \) is the minimum mortgage payment defined in Equation (5). The set of feasible solutions is characterized by

\[ a' \leq a' \leq y_j(z) + (1 + r) a - (1 + r_m) m + \bar{m}' - \delta ph - c \]

and

\[ \max \{ c + a' + \delta ph - y_j(z) - (1 + r) a, 0 \} \leq m' \leq \bar{m}'. \]
(iv) Use the budget constraint to find \( c_j^P(\omega_N) \). By definition, \( h_j^P(\omega_N) = h \), \( q_j^P(\omega_N) = q \), and \( r_{m,j}^P(\omega_N) = r_m \).

(d) Solving the problem of a borrower who defaults, \( V_j^D(\omega_D) \):

(i) This option is only available to existing borrowers (\( h > 0 \) and \( m > 0 \)).

(ii) Because a household who defaults must rent in the current period, use Equation (31) to substitute out \( s \) from the flow utility function and the budget constraint.

(iii) Solve for \( V_j^D(\omega_D) \) and \( a'_x j^R(\omega_D) \) using Brent’s method. The set of feasible solutions is characterized by

\[
 a' \leq a' \leq y_j(z) + (1 + r) a - \xi \left[ 1 + \left( \frac{1 - \alpha}{\alpha R^{1-\theta}} \right) \right]^{-1}.
\]

(iv) Back \( c_j^D(\omega_D) \) out from the flow budget constraint and use Equation (31) to find \( s_j^D(\omega_D) \).

(v) By definition, \( h_j^D(\omega_D) = 0 \) and \( m_j^D(\omega_D) = 0 \). Because the household does not have any mortgage debt, \( q_j^D(\omega_D) \) and \( r_{m,j}^D(\omega_D) \) can be set to any arbitrary value.

(e) Determine \( W_j^R(\omega_A) \) and \( W_j^O(\omega_N) \) using Equations (22) and (23).

(f) Compute \( \Pi_j(\omega_N) \) using Equation (29).

(g) Compute \( EW_j^R(\omega_A) \), \( EW_j^O(\omega_N) \), and \( E\Pi_j(\omega_N) \).

3. After solving for the stationary recursive equilibrium defined in Appendix B.1, interpolate value functions \( \{ V_j^R(\omega_N), V_j^M(\omega_N), V_j^P(\omega_N), V_j^D(\omega_N) \} \) and policy functions for endogenous state variables \( \{ a'_x j^R(\omega_N), h'_x j^R(\omega_N), m'_x j^R(\omega_N), q'_x j^R(\omega_N), r'_{m,j}^R(\omega_N) \} \) for all \( x \in \{ R, M, P, D \} \) over finer grids for liquid assets \( a \), mortgage debt \( m \), and the interest rate \( r_m \). Determine housing tenure and loan adjustment choices using the interpolated valued functions. Compute policy functions for the control variables accordingly.

4. Given the finer value functions and policy functions for endogenous state variables in the previous step, as well as the probability distributions of \( \delta \), \( p \), and \( z \), construct the \((ns_{fine}/T) \times (ns_{fine}/T)\) transition matrix for the distribution of households over states.

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74In order to preserve the non-convexities introduced by the discrete choices in this model, I interpolate \( V_j^M(\omega_N) \) and \( \psi_j(\omega_N) \) twice, first assuming the DTI limit applies and again assuming it does not, then apply the screening technology to determine if the no-DTI-limit option is feasible. Note that, in this step, I define finer policy and value functions over the originally defined state vector \( \omega_N \).
\( Q_j(\omega_N, \Omega_N) \) according to Equation (19), where \( (n_{s_{\text{fine}}}/T) \) denotes the number of fine grid points for a given \( j \).

5. For a given \( \mu_1(\omega_N) \) and \( Q_j(\omega_N, \Omega_N) \), use the law of motion in Equation (18) to compute \( \Lambda^*_j(\Omega_N) \) for all \( j \in \{1, 2, \ldots, T - 1\} \).