Default Risk Heterogeneity and Borrower Selection in the Mortgage Market*

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Abstract

This paper exploits variation from an institutional change in mortgage underwriting standards to document that the debt payment-to-income ratio is not a sufficient measure of default risk. As a result, households exhibit substantial heterogeneity in their ability to repay conditional on their debt payment-to-income ratios. To explain this, the paper studies an otherwise standard incomplete-markets life-cycle model with a competitive mortgage market in which lenders have discretion to exempt households whose equilibrium default probability satisfies a creditworthiness threshold from a debt payment-to-income limit. The model uniquely matches and rationalizes empirical patterns of default risk heterogeneity in the mortgage market. The model highlights welfare gains resulting from policy reforms that increase lender flexibility.

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1 Introduction

Since the Great Recession, mortgage underwriting standards have relied on the debt payment-to-income (DTI) ratio of borrowers as the preeminent measure of their ability to repay. The structural macroeconomic literature on endogenous mortgage default implies similar importance for it, as such models typically predict that default risk increases with the burden of debt service relative to income. In this paper, I exploit variation from an institutional change in underwriting standards allowing lenders to originate mortgages that exceed a statutory DTI limit to establish that the DTI ratio is not a sufficient measure of creditworthiness. To understand this, I introduce an endogenous loan contract choice and a tractable screening technology to an otherwise standard incomplete-markets life-cycle model with a competitive mortgage market. With the addition of these two features, the model is able to explain empirical features of default risk heterogeneity and borrower selection in the U.S. mortgage market that elude standard quantitative frameworks within the literature. Finally, I use the model to highlight scope for welfare-improving policy reform on which the current literature is silent.

Using data on conforming mortgage originations, I document that default risk exhibits a sharp discontinuity at the current DTI limit: relative to borrowers at the limit, those above it have higher credit scores and receive lower mortgage interest rates. Thus, even conditional on having large DTI ratios, borrowers still display substantial heterogeneity in their ability to repay. Additionally, I show that a fraction of borrowers on the DTI limit are no more likely to default than borrowers above it. These findings collectively indicate that borrowers with low default risk are selecting into loans with larger DTI ratios and that relatively creditworthy borrowers are likely constrained by the current limit.

To rationalize these facts, I study an incomplete-markets life-cycle model with competitive loan pricing and long-term illiquid mortgages à la Kaplan, Mitman and Violante (2020) that is augmented with two new features. The first, an endogenous loan contract choice, determines whether a DTI limit applies to the size of a new loan chosen by a household. This captures the fact that the limit binds for some, but not all, borrowers in the data. The second is a screening technology that requires households to satisfy a default risk threshold in order for a mortgage unconstrained by the DTI limit to be feasible. This feature reflects the discretion of lenders to exempt mortgages from this underwriting standard.

The calibrated model successfully matches the observed share of borrowers for whom the DTI limit binds; the non-monotonic relationship between default risk and DTI ratios; and the heterogeneity in default risk conditional on observed DTI ratios. The loan contract choice and screening technology generate borrower selection effects crucial for this result.
In the standard version of this model lacking these features, the equilibrium default risk of borrowers is monotonically increasing in their DTI ratio, and the variance of default risk across borrowers is counterfactually large. Of the households screened out of DTI-unconstrained loans by the default risk threshold, those with the highest default probabilities exit the mortgage market altogether, lowering the dispersion in default risk across borrowers in the model. The remainder reduce their desired loan size until their DTI ratios just satisfy the statutory maximum, allowing the model to replicate the observed discontinuity in default risk at the DTI limit and the heterogeneity across borrowers bunched below the limit.

The structural model explains why households with large desired DTI ratios are non-uniform in their underlying characteristics. Households constrained by the DTI limit along the extensive margin of debt are primarily existing renters who finance a new house purchase with a large loan. Their high equilibrium default risk reflects the low home equity they would possess. They are vulnerable to adverse shocks to cash on hand that could make future default optimal. By contrast, households constrained along the intensive margin are largely liquidity-constrained homeowners who refinance for precautionary reasons. Their comparatively lower default risk reflects their substantial positive home equity, which would allow them to liquidate their houses instead of default in response to negative shocks.

The degree of lender discretion permitted by mortgage underwriting standards is vital for evaluating the welfare implications of DTI limits. A model abstracting from this omits welfare gains that result from relaxing the limit precisely in states of the world in which households’ DTI ratios are likely to be large. To demonstrate this, I use the calibrated model to quantify the aggregate and distributional effects of a reform to the current limit proposed by the Dodd-Frank Act. The reform lowers the statutory limit and raises the implicit cost of originating loans above the limit while relaxing the associated creditworthiness standard. The trade-off for households is an expanded feasible set of DTI-unconstrained mortgages versus a higher equilibrium interest rate on such loans. I show that welfare gains from more lender flexibility in this reform are large, outweighing losses from higher interest rates.

The remainder of the paper proceeds as follows. In Section 2, I connect my paper to the relevant literature. In Section 3, I present empirical evidence on the selection of creditworthy borrowers into loans with large DTI ratios and the non-monotonicity of default risk with respect to the DTI ratio. In Section 4, I develop a theoretical model that explains these facts. In Section 5, I discuss the calibration strategy and validate model fit. In Section 6, I establish that the calibrated model can uniquely account for the patterns of loan selection and borrower heterogeneity documented in the data. In Section 7, I use the model to evaluate a proposed reform to the current DTI limit and show that the degree of lender discretion is important for understanding its effects on welfare. Section 8 concludes.
2 Related literature

On the theoretical side, my paper builds on a growing body of macroeconomic research that features uninsurable idiosyncratic risk, illiquid housing wealth, long-term debt, and borrowing constraints in models of the U.S. housing and mortgage markets.¹ My theoretical environment has much in common with models of household consumption that incorporate transaction costs to distinguish between liquid and illiquid assets.² With its inclusion of competitive loan pricing and endogenous default, my paper speaks to a related literature that uses equilibrium models to study bankruptcy and foreclosure.³

My empirical contribution is to document that a significant fraction of mortgage originations violate a statutory limit on debt payment-to-income ratios and that relatively more creditworthy borrowers hold such loans in equilibrium. By assumption, a model with a strict DTI limit—i.e., one that applies to a household’s loan size at origination without exception—cannot match these features of the data. Thus, my theoretical contribution is to embed in the standard framework an endogenous mortgage contract choice, a feature that remains relatively understudied in the literature.⁴ Borrowers in the model choose between DTI-constrained and -unconstrained loans. The choice is meaningful because it affects the expected payoffs of lenders and the discretion they have to relax the DTI limit.

My paper adds to ongoing research that studies the effects of *ex ante* limits on DTI ratios on the U.S. residential mortgage market.⁵ Because borrowers’ DTI ratios and default probabilities are equilibrium objects, my approach uses a calibrated structural model to conduct inference on unobservable household characteristics that explain the observed patterns of loan choice. My findings underscore the importance of accounting for the rich heterogeneity that exists even among households with similar desired DTI ratios and the contrasting ways in which constrained households respond to reforms to DTI limits. Policy

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³See Campbell and Cocco (2015), Chatterjee et al. (2007), and Mitman (2016). Some of the models mentioned in the first footnote also incorporate an endogenous foreclosure choice.

⁴Corbae and Quintin (2015) develop a model in which borrowers choose between competitively priced low- or high-down payment fixed-rate mortgages, but it places strong restrictions on the agents’ housing tenure and loan adjustment choices. Chambers, Garriga and Schlagenhauf (2009) study the implications of loan structure in general equilibrium but abstract from default risk. Gete and Zecchetto (2018) study an environment in which types of mortgages differ in the their down payment size and the credit-risk subsidy granted to the lender. However, they model mortgages as one-period debt without adjustment costs and abstract from a life-cycle savings motive.

⁵See Bhutta and Ringo (2015), DeFusco, Johnson and Mondragon (2020), and Greenwald (2018).
counterfactuals that abstract from this heterogeneity will therefore misstate the implications of such reforms for household welfare.

3 Empirical evidence on DTI ratios and default risk

Since 2009, the conforming mortgage market has operated under a limit of 45% on borrowers’ debt payment-to-income ratios.\(^6\) Despite this, a persistent share of mortgages continues to be originated with DTI ratios above this cap. Where previous work has focused on bunching at the statutory DTI limit as evidence of its influence on household leverage, I exploit its non-binding nature to document new stylized facts regarding the heterogeneity of default risk among borrowers conditional on their DTI ratios.

Prior to this policy, borrowers’ default risk was monotonically increasing in their DTI ratio. Afterwards, a discontinuity emerges at the 45% cutoff: relative to borrowers at the limit, borrowers above it have significantly higher credit scores and face lower costs of borrowing in equilibrium. Thus, in the region of the borrower distribution most directly affected by the DTI limit, a higher DTI ratio does not necessarily indicate a lower ability to repay. I also show that the distributions of credit scores associated with mortgages on either side of the limit overlap considerably. Some borrowers for whom the DTI requirement binds are therefore no more likely to default than those exempt from it, suggesting that relatively creditworthy households are constrained by the current policy.

3.1 Institutional background on DTI limits and data description

In March 2009, Freddie Mac revised its underwriting standards to include a 45% maximum back-end debt payment-to-income ratio at origination for manually underwritten mortgages.\(^7\) Freddie Mac is one of two large government-sponsored enterprises (GSEs) that purchase mortgage originations from primary lenders and issue securities backed by those mortgages to investors on the secondary market. Although Freddie Mac does not lend directly to borrowers themselves, lenders have strong incentives to originate mortgages that adhere to GSE underwriting standards. Since the Great Recession, the GSEs have jointly

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\(^6\)A conforming loan is a mortgage that satisfies the purchase requirements of Fannie Mae and Freddie Mac, two government-sponsored enterprises heavily involved in the secondary mortgage market.

\(^7\)Freddie Mac announced this change in a bulletin on November 24, 2008. The relevant policy is described in Chapter 5401 of the Freddie Mac Seller/Servicer Guide (see https://guide.freddiemac.com/app/guide/chapter/5401). The back-end DTI ratio includes in the numerator payments on all outstanding liabilities of the borrower—not only the first-lien mortgage—and housing expenses. Manual underwriting is the process by which a lender assesses various components of the loan application in accordance with underwriting requirements and risk evaluation guidelines published in the Freddie Mac Seller/Servicer Guide. Alternatively, a lender can use Freddie Mac’s proprietary automated underwriting software.
purchased or guaranteed around two-thirds of mortgage originations.\textsuperscript{8}

The underwriting standards specify extenuating circumstances under which lenders may originate a mortgage with a DTI ratio above 45% and still sell it to Freddie Mac. As a result, lenders have significant discretion to relax the DTI requirement. They include a borrower possessing enough liquid assets to constitute an ability to repay regardless of income; a down payment on the purchase of a property of at least 25%; or a strong credit score, defined as 740 or higher, combined with the lender’s written assurance that “the borrower’s credit reputation is excellent.” I will refer to loans that do and do not meet a statutory DTI limit as low-DTI loans and high-DTI loans, respectively, for the rest of the paper.

For the empirical analysis, I use the Freddie Mac Single Family Loan-Level Dataset. It contains quarterly loan-level data on fully amortized 15-, 20- and 30-year fixed-rate mortgage originations with full documentation that have been purchased or guaranteed by Freddie Mac since 1999. I limit my analysis to mortgages originated between 2005 and 2016 with 30-year terms that are collateralized by owner-occupied housing and have non-missing DTI ratios, credit scores, interest rates, and loan-to-value (LTV) ratios. This results in a sample of approximately 10 million loans. I provide descriptive statistics in Appendix A.1.

3.2 Selection of creditworthy borrowers into high-DTI loans

Although the share of mortgages with a debt payment-to-income ratio above 45% has declined relative to its pre-policy level, Figure 1 shows that the high-DTI category has accounted for around 9% of conforming loans since the DTI limit was introduced.\textsuperscript{9} This indicates that lenders do, in fact, exercise the discretion permitted by mortgage underwriting guidelines to originate high-DTI loans. The distribution of borrower default risk across DTI ratios has changed markedly over this time period. Before the addition of the DTI requirement, low- and high-DTI borrowers had average credit scores of 726 and 716, respectively.\textsuperscript{10} Although the average credit score of borrowers obtaining new loans has risen substantially since the Great Recession, high-DTI borrowers have become more creditworthy relative to their pre-policy level than low-DTI borrowers. Mean credit scores on either side of the 45% DTI limit are nearly identical under current regulations, suggesting that a borrower’s

\textsuperscript{8}Private-label securitization, which accounted for the plurality of loans originated during the housing boom, has represented less than 1% of originations volume over the same time period. See https://www.urban.org/sites/default/files/publication/102776/august-chartbook-2020.pdf.

\textsuperscript{9}In Appendix A.1, I plot the distribution of DTI ratios at origination by year. I also show that the timing and nature of changes in the distribution of DTI ratios are identical among loans purchased or guaranteed by Fannie Mae, the other GSE involved in the secondary market. I therefore conclude that the conforming loan market as a whole de facto operates under a 45% DTI limit.

\textsuperscript{10}This is the credit score of the borrower reported at the time of loan origination. It does not reflect any change to default risk that obtaining a new mortgage may induce.
DTI ratio has become less indicative of their default risk. Figure 2 confirms this intuition by showing that the DTI ratio is far from a sufficient proxy for default risk among borrowers with DTI ratios in the neighborhood of the 45% limit. In the left-hand panel, I plot residualized credit scores as a function of DTI ratios at origination for mortgages originated before and after the implementation of the DTI limit in 2009. Before the policy, a borrower’s credit score was monotonically decreasing in their DTI ratio, indicating that, in equilibrium, loans with larger DTI ratios were held by households with higher default risk. This correlation underlies common justifications for using DTI limits as a policy instrument for reducing household leverage and mortgage default. From 2009 onward, however, a clear discontinuity in credit scores emerges at the statutory maximum as high-DTI borrowers become more creditworthy. One way to contextualize this change is to note that borrowers with a DTI ratio of 50% have credit scores comparable to those of borrowers with a DTI ratio of 30%. To the left of the limit, though, credit scores remain strictly decreasing in the DTI ratio, and borrowers with DTI ratios exactly equal to 45% have the lowest credit scores of all. The non-monotonicity of default risk and DTI

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11I residualize with respect to a time trend in order to control for changes in aggregate economic conditions.
12For example, in justifying their regulatory focus on a borrower’s DTI ratio at origination, the Consumer Financial Protection Bureau (2013) writes, “...the Bureau believes that DTI is an indicator of the consumer’s ability to repay. All things being equal, consumers carrying loans with higher DTI ratios will be less able to absorb any such shocks and are more likely to default.”
ratios in the data shows that, in equilibrium, lenders only exercise the discretion they have to originate high-DTI loans in favor of borrowers with sufficient creditworthiness.

In the right-hand panel of Figure 2, I show that variation in \textit{ex ante} default risk is reflected in the interest rate that borrowers receive on their mortgages in equilibrium. Before 2009, the cost of borrowing is increasing in DTI ratios through the 45% cutoff. After 2009, it becomes discontinuous at this point: borrowers to the right of the DTI limit receive below-average interest rates, while those immediately to the left receive above-average interest rates. This finding is especially striking because a large DTI ratio can mechanically result from a high interest rate, all else equal.\footnote{A large DTI ratio can also, in theory, reflect a large initial loan size. In Figure 11 of Appendix A.1, I verify that the DTI ratios of borrowers to the right of the 45% limit are not being driven by their mortgages being especially large. After the DTI limit was introduced, high-DTI loans are associated with lower LTV ratios relative to loans just below the limit.}

The fact that DTI ratios above 45\% are associated with relatively lower interest rates further underscores the borrower selection effects at work.

To control for other observable variables that may affect these empirical patterns, I use a difference-in-differences specification common to this literature—e.g., DeFusco, Johnson and Mondragon (2020)—to estimate the change in the characteristics of a high-DTI loan, relative to a low-DTI loan, after the introduction of the 45\% DTI limit. The regression equation is

\[ y_{it} = \alpha + \beta_1 HighDTI_i + \beta_2 (HighDTI_i \times Policy_{it}) + \gamma_t + X_i' \delta + \varepsilon_{it}, \]  

\[ 1 \]
Table 1. Effect of the 45% DTI limit on default risk and interest rates at origination

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<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
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<tbody>
<tr>
<td></td>
<td>Credit score</td>
<td>Interest rate (%)</td>
</tr>
<tr>
<td>DTI &gt; 45%</td>
<td>-1.270***</td>
<td>0.002**</td>
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<tr>
<td></td>
<td>(0.000)</td>
<td>(0.023)</td>
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<tr>
<td>DTI &gt; 45% × Policy</td>
<td>10.705***</td>
<td>-0.055***</td>
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<td>(0.000)</td>
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<td>N</td>
<td>3,061,386</td>
<td>3,061,386</td>
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<td>$R^2$</td>
<td>0.139</td>
<td>0.909</td>
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p-level in parentheses. Robust standard errors are clustered at the state level. * p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The two rows contain estimated values for the coefficients $\beta_1$ and $\beta_2$ from the baseline difference-in-differences specification in Equation (1). Source: Freddie Mac Single Family Loan-Level Dataset.

where $y_{it}$ is a particular characteristic of loan $i$ originated in quarter $t$, $\alpha$ is a constant, $\gamma_t$ is a vector of quarter dummy variables, $X_i$ is a vector of loan-specific characteristics, and $\varepsilon_{it}$ is an error term clustered at the state level. $^{14} High_{DTI_i}$ is an indicator variable that takes a value of 1 if the DTI ratio of loan $i$ exceeds 45%. $Policy_{it}$ is an indicator variable that takes a value of 1 if the quarter of origination is 2009Q1 or later. $^{15}$ The coefficient of interest is $\beta_2$, which represents the differential change in the dependent variable for high-DTI loans relative to low-DTI loans after the introduction of the 45% DTI limit. When estimating the regression, I restrict my sample to loans with a DTI ratio between 40% and 50%. $^{16}$

I estimate the regression using the credit score and interest rate as dependent variables. Table 1 displays estimated values for $\beta_1$ and $\beta_2$ from Equation (1). The correlations documented in Figure 2 survive after controlling for this richer set of covariates and are all statistically significant at the 1% level. Relative to borrowers with low-DTI loans, the credit score of borrowers with high-DTI loans increases by 10.7 after the DTI limit is introduced. The interest rate they receive on their loans correspondingly declines by 5.5 basis points. As a robustness check, I consider in Appendix A.2 a flexible difference-in-differences specification that allows the effect of the limit to vary with the DTI ratio. Results from that specification

$^{14}$ $X_i$ includes dummy variables for the state in which the property is located, loan purpose (i.e., purchase or refinance), type of property, number of units on the property, and whether the borrower is a first-time homebuyer.

$^{15}$ Including $Policy_{it}$ as a separate regressor is unnecessary because it is absorbed by the quarter dummy variables.

$^{16}$ The results are robust to estimating the regression equation for the full sample of loans.
confirm that the effects I have documented here are driven by a change exactly at the 45\% DTI limit.

3.3 Heterogeneity in default risk among borrowers on the DTI limit

The findings from Section 3.2 show that lenders exercise the discretion they have to originate high-DTI loans only for borrowers whose ability to repay is beyond doubt. The DTI limit, however, still applies to the loan choice of borrowers who fall short of this creditworthiness standard. As a result, the borrowers for whom the DTI limit appears most binding—i.e., those with DTI ratios just at or below 45\%—exhibit the highest \textit{ex ante} default risk on average.

Next, I document that, despite this difference in mean default risk on either side of the DTI limit, borrowers at the limit are not uniform in their ability to repay. In Figure 3, I plot distributions of credit scores associated with mortgages originated in 2009Q1 and later separately for borrowers with DTI ratios above 45\% and those with DTI ratios between 43\% and 45\%.\textsuperscript{17} It reveals the existence of significant heterogeneity in the default risk across borrowers with DTI ratios at the statutory limit. The credit scores of high-DTI borrowers display lower variance. The latter is expected, given that lenders can only justify high-DTI loans for households whose ability to repay is assured.

Additionally, Figure 3 provides suggestive evidence that a nontrivial share of borrowers located at the DTI limit are constrained by it despite not being observationally more likely to default than borrowers who met the creditworthiness standard for high-DTI loans. This can be seen by looking at the share of borrowers at the limit who have credit scores of 740 or above. Since the current limit was implemented in 2009, around 20\% of mortgage originations with DTI ratios between 43\% and 45\% have gone to borrowers with credit scores of at least 740.\textsuperscript{18} This cutoff is relevant because, as discussed in Section 3.1, one condition under which a lender may justify waiving the DTI requirement for a prospective borrower is if they have a credit score of at least 740.\textsuperscript{18} The data make clear that credit score in this range is neither a necessary nor sufficient condition for obtaining a high-DTI loan. Nonetheless, it is an observable proxy for the creditworthiness threshold that a borrower must meet in

\textsuperscript{17}43\% is the maximum DTI ratio specified by the Dodd-Frank ability-to-repay rule and will be relevant in the policy counterfactual in Section 7. Restricting the comparison to borrowers who have a DTI ratio equal to 45\% results produces nearly identical results.

\textsuperscript{18}To place this in perspective, a credit score between 740 and 799 indicates “very good” credit. It is also relevant to lenders who sell to Freddie Mac. Each loan sold to Freddie Mac is charged a delivery fee that depends on the credit score and LTV ratio of the borrower. Conditional on the LTV ratio, delivery fees are decreasing in the credit score, with 740 representing the top tier of creditworthiness. See \url{https://www.experian.com/blogs/ask-experian/infographic-what-are-the-different-scoring-ranges/} and \url{http://www.freddiemac.com/singlefamily/pdf/ex19.pdf}, respectively.

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order to qualify for a high-DTI loan.

Altogether, these stylized facts highlight the heterogeneity in default risk that exists among borrowers with large DTI ratios. Relatively more creditworthy borrowers select into high-DTI mortgages, whereas borrowers located at the statutory limit represent a more heterogeneous mix of default probabilities. These observations raise further questions. First, why do households who vary in their ability to repay have similar desired DTI ratios in equilibrium? Second, how binding are the current DTI limit and the amount of discretion lenders have to relax it with respect to household allocations? Understanding this is vital for assessing the implications of DTI limits and requires identifying the unobserved household heterogeneity that generates these equilibrium outcomes in the mortgage market. In the following sections, I develop a theoretical framework to rationalize the features of the data documented above and answer these questions.

4 A model of mortgage contract choice and lender discretion

To explain the novel aspects of the data documented in Section 3, I introduce two features to an otherwise standard incomplete-markets life-cycle model with a competitive mortgage market. Motivated by the existence of borrowers with debt payment-to-income ratios at
origination that exceed the statutory maximum, I embed in the model an *endogenous loan contract choice* in which households entering into new loans have an option between a low- or high-DTI mortgage. This feature is needed to generate a distribution of households in which a DTI limit binds for some borrowers but not all.

For this distinction between loan types to be theoretically meaningful, an additional friction is needed such that some households for whom a high-DTI loan can be sustained in equilibrium are unable to obtain one. I therefore introduce a *loan screening technology* in the form of a default risk threshold that households must satisfy in order for a high-DTI loan to be feasible. In the standard framework used by the literature, competitive financial intermediaries are risk neutral and have perfect information. As a result, the market for loans originated to households in their current state clears automatically as long as a zero-expected profit condition holds in equilibrium. The default risk threshold is a tractable way to model the discretion that lenders in reality have to relax the DTI requirement for borrowers deemed sufficiently creditworthy. One of the contributions of the model is to quantify how much discretion lenders have under current underwriting standards.

In the rest of this section, I summarize the model environment with particular emphasis on the loan contract choice and screening technology. I then specify the optimization problems of households and lenders. Lastly, I define a stationary recursive equilibrium of the model.

### 4.1 Model overview

The model features a constant population of overlapping generations of households who split their lives between working and retirement. When working, they receive an age-specific endowment income subject to uninsurable idiosyncratic risk. When retired, they receive a constant pension income. Households can save in both a liquid asset and illiquid housing wealth. Homeowners can borrow through illiquid long-term, fixed-rate mortgage contracts subject to borrowing limits at origination. Debtors have the option to default. Households derive utility each period from consuming a nondurable good and housing services. Housing services are obtained by either renting or purchasing a house that yields a service flow each period. House prices are subject to uninsurable idiosyncratic shocks. Owner-occupied houses are also subject to depreciation shocks.

The model also features a continuum of competitive, risk-neutral, and infinitely lived financial intermediaries. They store the liquid savings of and supply mortgage debt to households. Because financial intermediaries observe the household’s idiosyncratic state, the equilibrium interest rate on a newly originated mortgage reflects the household’s endogenous default risk and is such that the lender makes zero expected profit on a loan-by-loan basis.
Preferences and endowments

A household maximizes expected discounted lifetime utility, defined as

$$\max \mathbb{E} \left\{ \sum_{j=1}^{T} [\beta^{j-1} u_j (c_j, s_j)] + \beta^T \nu (W_{T+1}) \right\},$$

where $j \in \{1, 2, \ldots, T\}$ is the household’s age, $\beta \in (0, 1)$ is the subjective discount factor, and $u (\cdot)$ is the flow utility function. $u (\cdot)$ satisfies the Inada conditions and is given by

$$u_j (c_j, s_j) = \frac{1}{1 - \sigma} \left[ \alpha_j c_j^{1-\theta} + (1 - \alpha_j) s_j^{1-\vartheta} \right]^{\frac{1-\sigma}{1-\vartheta}},$$

where $c$ is nondurable consumption and $s$ is housing services. $\sigma > 0$ is the coefficient of relative risk aversion, and $1/\vartheta > 0$ is the elasticity of substitution between nondurable consumption and housing services. $\alpha_j \in (0, 1)$ is the preference weight on nondurable consumption that potentially depends on age. Households have a bequest motive such that they receive expected discounted utility from end-of-life wealth $W_{T+1}$ according to the function

$$\nu (W_{T+1}) = B \mathbb{E} \left\{ \frac{W_{T+1}^{1-\sigma}}{1 - \sigma} \right\}.$$

The parameter $B > 0$ controls the strength of the bequest motive. This is needed to ensure that, as in the data, households reach the end of their lives with positive net worth.

A household supplies labor inelastically from age 1 until they retire at age $T_R$. While working, a household’s log income is the sum of a deterministic component indexed by age $\chi_j$ and an idiosyncratic component $z$ that evolves according to the first order Markov process

$$z' = \rho z + \varepsilon', \quad \varepsilon' \sim i.i.d. \mathcal{N} (0, \sigma_z^2).$$

Upon retirement, a household receives a constant pension that is a deterministic function of earnings in the last year of their working life. The process for log income is therefore

$$\log y_j (z) = \begin{cases} \chi_j + z & \text{if } 1 \leq j \leq T_R - 1 \\ \Phi (y_{T_R-1} (z)) & \text{if } T_R \leq j \leq T. \end{cases}$$

19This captures life cycle-related components of demand for housing services, such as changes in family size, that are not explicitly modeled. The calibration will imply that retired households have a lower preference weight on housing services than working households.
Liquid savings

All households can save in a one-period liquid asset $a$ that is subject to a no-borrowing constraint. The liquid asset takes the form of deposits held by the financial intermediaries. The intermediaries have access to international capital markets where the risk-free rate $r$ is determined by the net supply of safe financial assets. A zero-profit condition on deposits implies that the household also earns the risk-free rate on their liquid savings.

Housing

Housing services are obtained through either the rental or owner-occupied housing market. If the household participates in the owner-occupied housing market, they purchase a house of size $h$ at price $p$. They choose a house size from a discrete grid and can only own one house at a time.\textsuperscript{20} The supply of houses of all sizes is perfectly elastic. The house yields a one-to-one flow of services (i.e., $s = h$) each period. Following Berger et al. (2017) and Mitman (2016), house prices are subject to an idiosyncratic shock and follow the first order Markov process

$$\log p' = \rho \log p + \eta', \quad \eta' \sim \mathcal{N}(0, \sigma^2 \eta).$$

The house price affects the value of the house but not the quantity of services it generates. Households who adjust their housing stock pay a fixed transaction cost $\kappa_h > 0$. As in Chatterjee and Eyigungor (2015), owner-occupied housing is subject to an i.i.d. depreciation shock $\delta \in (0, 1)$ each period, where

$$\delta = \begin{cases} \bar{\delta} & \text{with probability } \zeta \\ \hat{\delta} & \text{with probability } 1 - \zeta \end{cases}$$

and $\bar{\delta} > \hat{\delta}$. Owners pay a housing maintenance cost of $\delta ph$ each period.

The supply of rental housing is perfectly elastic such that a household can rent $s$ units of housing services at a rate $Rp$ per unit each period, where $R > 0$ denotes the exogenous rent-price ratio.\textsuperscript{21} Adjusting the quantity of rental housing between periods is costless, and renters do not experience depreciation shocks.

\textsuperscript{20} This captures the indivisibility of housing as an asset, emphasized by Piazzesi and Schneider (2016).
\textsuperscript{21} The quantitative results are robust to the assumption of a constant rent-price ratio.
Mortgages

Owners can use their house as collateral for fixed-rate mortgage debt \( m \) that is amortized over their remaining lifetime.\(^{22}\) They borrow at an equilibrium interest rate \( r_m \) that is a function of their idiosyncratic state. When obtaining a new loan, households choose a type of mortgage contract represented by \( q \in \{ L, H \} \), where \( L \) and \( H \) denote a low- and high-DTI loan, respectively. The current state of an age-\( j \) household is therefore summarized by the vector \( \omega \equiv (a, h, \delta, p, m, q, r_m, z) \).\(^{23}\)

For a high-DTI loan to be feasible, a household must satisfy a default risk threshold. Let \( \psi_j(\omega) \) denote a household’s endogenous probability of default and \( \Psi \in (0, 1) \) denote the default risk threshold. The threshold is a primitive of the model and represents the amount of discretion that lenders have to originate high-DTI loans.\(^{24}\) It tractably introduces a screening problem into the model and generates households who have similar levels of default risk but select different mortgage contracts in equilibrium.\(^{25}\) By contrast, a household who chooses a low-DTI loan does not need to satisfy a default risk requirement for it to be feasible, conditional on it being supported in equilibrium.

The novel formulation of the DTI limit highlights the effects of loan contract choice and the screening technology on how much the household can borrow. The DTI limit,

\[
\pi_{\text{min},j}(m, r_m) \leq \lambda(q) y_j(z),
\]

states that the minimum mortgage payment \( \pi_{\text{min},j}(m, r_m) \) cannot exceed a fraction \( \lambda(q) \) of the household’s contemporaneous income \( y_j(z) \) at origination. Because the contract choice determines if the DTI limit applies and the household’s default risk determines if the high-

\(^{22}\)This implies that the maturity of a newly originated mortgage is \( T - j \). I follow other papers in the literature like Kaplan, Mitman and Violante (2020) and Wong (2021) in making loan maturity a function of the household’s age. It is consistent with the observed negative correlation between age and loan duration and ensures that I need not track loan maturity as an additional state variable.

\(^{23}\)The contract type is tracked as a state variable because it may potentially affect the expected present value of a new mortgage through the lender’s payoff from default. In the baseline calibration, this payoff is independent of contract type. I relax this assumption in my policy counterfactual in Section 7.

\(^{24}\)The automatic denial of high-DTI loans to households whose credit risk is too high can be interpreted through the lens of automated—as opposed to manual—underwriting. For example, lenders can use Freddie Mac’s proprietary software to underwrite loans. If it returns a result stating that the loan would be ineligible for sale to Freddie Mac, then it is unlikely that the lender will originate a mortgage on those terms. The default risk threshold is also isomorphic to an exogenous constraint on credit supply studied by Justiniano, Primiceri and Tambalotti (2019).

\(^{25}\)In general, this literature abstracts from asymmetric information in lending markets. One exception is Chatterjee et al. (2020), who study a model of bankruptcy with in which households have hidden types and lenders update their beliefs about borrower type.
DTI contract is feasible, I specify $\lambda (q)$ as

$$
\lambda (q) = \begin{cases} 
\lambda - \varsigma_j (\omega) & \text{if } q = L \\
\infty & \text{if } q = H \text{ and } \psi_j (\omega) \leq \Psi.
\end{cases}
$$

(4)

The choice of a low-DTI loan constrains the DTI ratio of a mortgage to be less than $\lambda - \varsigma_j (\omega)$, where $\lambda$ is the statutory maximum DTI ratio and $\varsigma_j (\omega)$ is an exogenous offset term that represents non-first mortgage liabilities and housing expenses relative to income. Following the existing literature, I adjust the statutory limit in the model downward in order to bring DTI ratios in line with the data because mortgage underwriting standards target a borrower’s back-end DTI ratio.\textsuperscript{26} Where previous papers have assumed a uniform DTI offset, I instead model it as a deterministic function of the household’s state. The choice of a high-DTI loan completely relaxes this borrowing constraint but is only available to households who are sufficiently creditworthy in equilibrium.

The other features of mortgage debt are standard. In addition to the DTI constraint, a loan is subject to the usual loan-to-value constraint at origination,

$$m \leq \theta ph,$$

which limits the loan size to a fraction $\theta \in (0, 1)$ of the value of the home $ph$. A household who obtains a new mortgage pays a fixed transaction cost $\kappa_m > 0$, and a borrower can only hold one loan at a time. An age-$j$ borrower with predetermined debt $m$ and interest rate $r_m$ makes a mortgage payment of at least $\pi_{\text{min},j} (m, r_m)$ each period such that the outstanding balance on the loan evolves according to

$$m' \leq (1 + r_m) m - \pi_{\text{min},j} (m, r_m),$$

where $\pi_{\text{min},j} (m, r_m)$ is computed with the amortization formula

$$\pi_{\text{min},j} (m, r_m) = \frac{(1 + r_m)^{T-(j-1)}}{(1 + r_m)^{T-1} - 1} r_m m. \quad (5)$$

\textsuperscript{26}See, for example, Boar, Gorea and Midrigan (2020) and Greenwald (2018).
\((1 - \delta) ph - \gamma(q)\), where \(\gamma(q)\) represents resource costs of foreclosure and is given by

\[
\gamma(q) = \begin{cases} 
\gamma_L & \text{if } q = L \\
\gamma_H & \text{if } q = H.
\end{cases}
\] (6)

The parameters \(\gamma_L > 0\) and \(\gamma_H > 0\) allow the foreclosure cost to potentially vary across high- and low-DTI loans.\(^{27}\)

4.2 Household’s optimization problem

The household’s optimization problem is written recursively. Denote the expected discounted lifetime utility of an age-\(j\) renter in state \(\omega\) by

\[
W^R_j(\omega) = \max_{x_j(\omega)} \{ V^R_j(\omega), V^M_j(\omega) \}. \] (7)

Denote the expected discounted lifetime utility of an age-\(j\) homeowner in state \(\omega\) by

\[
W^O_j(\omega) = \max_{x_j(\omega)} \{ V^R_j(\omega), V^M_j(\omega), V^P_j(\omega), V^D_j(\omega) \}. \] (8)

The value functions inside the maximum operators of Equations (7) and (8) correspond to the discrete choices over housing tenure and loan adjustment available to a household in their current state. \(V^R_j(\omega)\) is the value of renting; \(V^M_j(\omega)\) is the value of owning and obtaining a new mortgage loan; \(V^P_j(\omega)\) is the value of owning and making a payment on an existing loan; and \(V^D_j(\omega)\) is the value of defaulting on outstanding debt. \(x_j(\omega) \in \{R, M, P, D\}\) denotes the decision to rent, obtain a new mortgage, continue with an existing loan, and default, respectively. Figure 4 summarizes these choices.

The timing of the model is as follows. At the beginning of the period, a household receives the realizations of their idiosyncratic shocks. A household decides among the housing tenure and loan adjustment options available to them by solving the associated optimization problems and selecting the choice that yields the highest expected lifetime utility. Consumption occurs at the end of the period.

\(^{27}\)These parameters stand in for legal costs associated with judicial foreclosures, administrative overhead associated with acquiring a foreclosed property, and lack of maintenance that results from the property remaining unoccupied. Pennington-Cross (2006) and Campbell, Giglio and Pathak (2011) find that the property values of foreclosed houses tends to be lower than those of non-foreclosed houses.
Conditional on renting, a household chooses nondurable consumption, rental housing, and liquid savings to solve

\[
V^R_j(\omega) = \max_{c,s,a'} u_j(c,s) + \beta \mathbb{E}_{\delta',p',z'|p,z} W^R_{j+1}(\omega') \\
\text{s.t.} \\
c + Rps + a' \leq y_j(z) + (1 + r) a + (1 - \delta) ph - (1 + r_m) m - 1_{h' \neq 0} \kappa_h \\
a' \geq 0 \\
\omega' = (a', 0, \delta', p', 0, 0, 0, z').
\]

(9)

Conditional on owning and obtaining a new mortgage, a household chooses nondurable consumption, liquid savings, house size, mortgage debt, and mortgage contract type to solve

\[
V^M_j(\omega) = \max_{c,a',h',m',q' \in \{L,H\}} u_j(c,h') + \beta \mathbb{E}_{\delta',p',z'|p,z} W^O_{j+1}(\omega') \\
\text{s.t.} \\
c + a' + ph' \leq y_j(z) + (1 + r) a + (1 - \delta) ph - (1 + r_m) m + m' - 1_{h' \neq h} \kappa_h - 1_{m' > 0} \kappa_m \\
m' \leq \theta ph' \\
\pi_{\min,j}(m', r'_{m,j}(\omega)) \leq \lambda_j(q') y_j(z) \\
\lambda_j(q') = \begin{cases} \\
\lambda - \zeta_j(\omega) & \text{if } q' = L \\
\infty & \text{if } q' = H \text{ and } \psi_j(\omega) \leq \Psi \\
\end{cases} \\
a' \geq 0 \\
\omega' = (a', h', \delta', p', m', q', r'_{m,j}(\omega), z').
\]

(10)
Here, I make explicit the interaction of households with the mortgage market when they enter into a new loan.\textsuperscript{28} The interest rate that a household receives on a new loan $r'_{m,j}(\omega)$ is determined in equilibrium and carried into tomorrow’s state $\omega'$ because it is fixed for the duration of the mortgage. Likewise, the contract type chosen by the household $q'$ appears in $\omega'$ because it affects the lender’s payoff in the event of future default. The endogenous default risk used to qualify a household for a high-DTI loan is defined as

$$
\psi_j(\omega) \equiv \mathbb{E}_{\delta', p', z'|p, z} \{ \mathbbm{1}_{x_{j+1}(\omega')=D|x_j(\omega)=M} \text{ and } q'_j(\omega)=H \},
$$

i.e., the probability that an age-$j$ household in state $\omega$ who chooses a high-DTI mortgage optimally defaults at age $j + 1$.

Conditional on continuing with an existing loan, an owner chooses nondurable consumption, liquid savings, and a mortgage payment to solve

$$
V_j^P(\omega) = \max_{c, a', m'} u_j(c, h) + \beta \mathbb{E}_{\delta', p', z'|p, z} W_{j+1}^O(\omega')
$$

s.t.

$$
c + \delta p h + a' \leq y_j(z) + (1 + r) a - (1 + r_m) m + m'
$$

$$
m' \leq (1 + r_m) m - \pi_{min,j}(m, r_m)
$$

$$
a' \geq 0
$$

$$
\omega' = (a', h, \delta', p', m', q, r_m, z').
$$

Note that, in this case, the borrower’s state in the next period reflects the fact that they continue with their predetermined house size, contract type, and mortgage interest rate.

Conditional on defaulting, an owner with outstanding debt chooses nondurable consumption, rental housing, and liquid savings to solve

$$
V_j^D(\omega) = \max_{c, s, a'} u_j(c, s) - \xi + \beta \mathbb{E}_{\delta', p', z'|p, z} [\varphi W_{j+1}^R(\omega') + (1 - \varphi) V_{j+1}^R(\omega')]
$$

s.t.

$$
c + R p s + a' \leq y_j(z) + (1 + r) a
$$

$$
a' \geq 0
$$

$$
\omega' = (a', 0, \delta', p', 0, 0, 0, z').
$$

\textsuperscript{28}The indicator variable on the loan transaction cost $\kappa_m$ accounts for the possibility that households with high net worth and/or income may optimally adjust their house size without simultaneously obtaining a new mortgage. Such households only account for 0.35% of the population in the stationary distribution of the calibrated model.
Because a defaulting household discharges existing debt and loses their house, they do not cover the minimum mortgage payment or housing maintenance costs that would otherwise be due. They incur a flow utility loss $\xi > 0$ in the period of default and are excluded from the owner-occupied housing and mortgage markets for a stochastic period of time. With exogenous probability $\varphi$, they regain access to these markets in the next period whereas, with probability $1 - \varphi$, they must continue renting.

At age $T$, the household is prohibited from further borrowing. This imposes the restriction that $m' = 0$ on the optimization problems in Equations (10) and (12). End-of-life wealth consists of liquid savings and, if applicable, the value of the home net of depreciation such that $W_{T+1} = (1 + r) a' + (1 - \delta') p'h'$.

### 4.3 Financial intermediary’s optimization problem

Mortgage contracts are issued by financial intermediaries who maximize expected profit. Intermediaries are risk neutral, infinitely lived, and perfectly competitive. They have perfect information and can observe a household’s current state and decision rules. They discount the future at rate $r + \phi$, where $\phi > 0$ is a parameter that captures mortgage servicing costs.\(^{29}\)

The present value of an existing mortgage held by an age-$j$ household in state $\omega$ is

$$
\Pi_j (\omega) = \begin{cases} 
(1 + r_m) m & \text{if repay} \\
(1 - \delta) ph - \gamma (q) & \text{if default} \\
(1 + r_m) m - m_j' (\omega) + \frac{1}{1 + r + \phi} \mathbb{E}_{\delta', p', z'} | p, z \Pi_{j+1} (\omega') & \text{otherwise,}
\end{cases}
$$

(14)

where $\omega' = (a_j' (\omega), h_j' (\omega), \delta', p', m_j' (\omega), q_j' (\omega), r_{m, j}' (\omega), z')$ reflects the optimal choices of households in their current state. If the borrower repays the loan, then the intermediary receives the remaining balance plus interest. If they default, then the intermediary recovers the value of the house posted as collateral net of depreciation and foreclosure costs. If they continue with the loan, then the intermediary receives their mortgage payment and the continuation value of the loan.\(^{30}\)

\(^{29}\)These are direct costs associated with payment collection, quality assurance, or corporate overhead. See https://www.mba.org/publications/insights/archive/mba-insights-archive/2018/mba-chart-of-the-week-average-servicing-costs-per-loan.

\(^{30}\)Requiring a borrower to either repay or default on outstanding debt at the end of life implies that

$$
\Pi_T (\omega) = \begin{cases} 
(1 + r_m) m & \text{if repay} \\
(1 - \delta) ph - \gamma (q) & \text{if default.}
\end{cases}
$$

Given that $\Pi_T (\omega)$ is a deterministic function of the household’s state, this makes it possible to solve the model numerically through backward induction.
On a loan-by-loan basis, the face value of a newly originated loan equals the expected present value of its future cash flows in equilibrium. Thus, conditional on the household’s current idiosyncratic state, the financial intermediary offers an interest rate $r'_{mj}(\omega)$ that satisfies the zero-expected profit condition

$$m'_j(\omega) = \frac{1}{1 + r + \phi} \mathbb{E}_{\omega' \mid \omega, z'} [\Pi_{j+1}(\omega')].$$

(15)

4.4 Equilibrium definition

I solve for the stationary recursive equilibrium of the model. To establish notation, I define the state space, holding age $j$ fixed, as $W \equiv A \times H \times D \times P \times M \times Q \times R_M \times Z$. Let the $\sigma$-algebra $\Sigma_W$ be defined as $B_A \otimes B_H \otimes B_M \otimes B_Q \otimes B_{RM} \otimes P(D) \otimes P(P) \otimes P(Z)$, where $B_A$, $B_H$, $B_M$, $B_Q$, and $B_{RM}$ are the Borel $\sigma$-algebras on $A$, $H$, $M$, $Q$, and $R_M$, respectively, and $P(D)$, $P(P)$, and $P(Z)$ are the power sets of $D$, $P$, and $Z$, respectively. Define $\Omega \equiv A \times H \times D \times P \times M \times Q \times R_M \times Z$ as the typical subset of $\Sigma_W$.

For a given parameterization of the model and a measure of age-1 households $\mu_1(\omega)$, a stationary recursive equilibrium consists of

1. household value functions $\{V^R_j(\omega), V^M_j(\omega), V^P_j(\omega), V^D_j(\omega)\}$ and policy functions $\{c_j(\omega), s_j(\omega), a'_j(\omega), h'_j(\omega), m'_j(\omega), q'_j(\omega)\}$;

2. a mortgage interest rate schedule $r'_{mj}(\omega)$; and

3. a stationary measure $\Lambda^*_j(\Omega)$;

such that,

1. given $r'_{mj}(\omega)$, household value and policy functions solve the optimization problems in Equations (7), (8), (9), (10), (12), and (13);

2. given the household’s value and policy functions, $r'_{mj}(\omega)$ is such that financial intermediaries’ zero-expected profit condition in Equation (15) is satisfied on a loan-by-loan basis; and

3. the invariant probability measure satisfies

$$\Lambda^*_{j+1}(\Omega) = \int_{\Omega} Q_j(\omega, \Omega) \left[ \Lambda^*_{j}(d\omega) + \mu_1(d\omega) \right]$$

(16)
for all $\Omega \in \Sigma_W$, where $Q_j(\omega, \Omega)$ is the conditional probability that an age-$j$ household in state $\omega$ transitions to the set $\Omega$ at age $j + 1$ and defined as

$$Q_j(\omega, \Omega) \equiv 1_{a'_j(\omega) \in A, h'_j(\omega) \in H, m'_j(\omega) \in M, a'_j(\omega) \in Q, r'_{m,j}(\omega) \in R_M, \sum_{\delta' \in D} \sum_{p' \in P} \sum_{z' \in Z} \pi(\delta') \pi(p'|p) \pi(z'|z).$$

(17)

I solve the model numerically using backward induction and provide a detailed description of the solution algorithm in Appendix C.

5 Calibration

The goal of the calibration is to assure that the stationary distribution of households in the model matches not only salient features of household balance sheets but also mortgage market outcomes under the current 45% debt payment-to-income limit documented in Section 3. In particular, by requiring the calibrated model to match the observed shares of borrowers found above and just below the limit, I quantify how much the constraint binds for households. I do not target the discontinuity in default risk at the DTI limit or the fraction of creditworthy borrowers on the limit. I instead use these outcomes to validate the mechanism in the model.

Where possible, I assign parameter values directly by relying on direct empirical evidence or standard values in the literature. The remaining parameters are calibrated by minimizing the weighted distance between a set of empirical moments and their model equivalents. After discussing these two sets of parameters separately, I show that the calibrated model does well in matching the life-cycle profiles of household balance sheets and consumption, along with aggregate moments regarding homeownership, mortgage debt, and default.

5.1 Externally calibrated parameters

Table 2 lists the directly assigned parameters and their sources.

Preferences and endowments

One period in the model corresponds to one year. Households enter the model at age 25, retire at age 65, and die at age 80, implying $T = 56$ and $T_R = 41$.\textsuperscript{31} The inverse elasticity of substitution between nondurable consumption and housing services $\psi$ is 0.80.\textsuperscript{32} The coefficient of relative risk aversion $\sigma$ is 2, a standard value in the macroeconomics literature.

\textsuperscript{31}As in Berger et al. (2017), I set the initial age of households in the model to 25 to avoid complications arising from schooling decisions.

\textsuperscript{32}See Piazzesi, Schneider and Tuzel (2007).
Table 2. Externally assigned parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_j$ Age-specific income</td>
<td></td>
<td>PSID (1999–2017)</td>
</tr>
<tr>
<td>$\mu_1$ Distribution of age-1 households</td>
<td></td>
<td>SCF (2016)</td>
</tr>
<tr>
<td>$\Phi$ Pension income</td>
<td></td>
<td>Guvenen and Smith (2014)</td>
</tr>
<tr>
<td>$\varsigma_j$ DTI constraint offset</td>
<td></td>
<td>SCF (2016) and AHS (2017)</td>
</tr>
<tr>
<td>$\delta$ Low depreciation rate (%)</td>
<td>1.064</td>
<td>PSID (2005–2017)</td>
</tr>
<tr>
<td>$\lambda$ Statutory DTI limit (%)</td>
<td>45</td>
<td>Freddie Mac</td>
</tr>
<tr>
<td>$\varphi$ Exclusion from mortgage market</td>
<td>0.143</td>
<td>Experian</td>
</tr>
<tr>
<td>$r$ Risk-free rate (%)</td>
<td>1.270</td>
<td>FRED (1971–2016)</td>
</tr>
<tr>
<td>$\rho_p$ Persistence of house price shock</td>
<td>0.970</td>
<td>Mitman (2016)</td>
</tr>
<tr>
<td>$\rho_z$ Persistence of income shock</td>
<td>0.977</td>
<td>Storesletten, Telmer and Yaron (2004)</td>
</tr>
<tr>
<td>$\sigma$ Risk aversion</td>
<td>2</td>
<td>Standard in literature</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$ Std. dev. of income shock</td>
<td>0.155</td>
<td>Storesletten, Telmer and Yaron (2004)</td>
</tr>
<tr>
<td>$\sigma_\eta$ Std. dev. of house price shock</td>
<td>0.080</td>
<td>Mitman (2016)</td>
</tr>
<tr>
<td>$T$ Number of model periods</td>
<td>56</td>
<td>U.S. life expectancy of age 80</td>
</tr>
<tr>
<td>$T_R$ Retirement age</td>
<td>41</td>
<td>Retirement at age 65</td>
</tr>
<tr>
<td>$\theta$ Statutory LTV limit (%)</td>
<td>85</td>
<td>Greenwald (2018)</td>
</tr>
<tr>
<td>$\vartheta$ Inv. elasticity of sub. btw. $c$ and $s$</td>
<td>0.800</td>
<td>Piazzesi, Schneider and Tuzel (2007)</td>
</tr>
</tbody>
</table>

Following Storesletten, Telmer and Yaron (2004), I set the persistence of shocks to the idiosyncratic component of income $\rho_z$ to 0.977 and its standard deviation $\sigma_\varepsilon$ to 0.155. Following Kaplan and Violante (2014), I use data from the 1999–2017 waves of the Panel Study of Income Dynamics (PSID) to estimate the deterministic age-dependent component of income. I parameterize the function for pension income using the procedure described by Guvenen and Smith (2014).\(^{33}\)

Liquid savings

The risk-free rate $r$ is set to 1.27%, the difference between the 1-year Treasury constant maturity rate and annual CPI inflation averaged over the years 1971–2016.

Housing

Following Mitman (2016), I set the persistence of the idiosyncratic house price shock $\rho_p$ to 0.970 and its standard deviation $\sigma_\eta$ to 0.080. I set the low depreciation rate $\delta$ to 1.06% in order to match the mean ratio of annual home maintenance expenditures to the value of the primary residence from the PSID.\(^{34}\)

\(^{33}\)See Appendix B.1 for additional details.

\(^{34}\)Specifically, I use question F87, “How much did you (and your family living there) spend altogether in [previous year] on home repairs and maintenance, including materials plus any costs for hiring a professional?” This question was added to the PSID in 2005 and is distinct from question W69, which asks households to only report home improvements worth at least $10,000.
Mortgages

The statutory maximum DTI ratio \( \lambda \) is 45%. I parameterize the DTI offset function \( \varsigma_j(\omega) \) by regression household balance sheet and demographic variables on non-mortgage DTI ratios using data from the 2016 Survey of Consumer Finances (SCF). The estimated coefficients are then used to compute non-mortgage DTI ratios for borrowers in the model given their age and current state. I add to that the mean ratio of housing expenses relative to income, calculated using the 2017 American Housing Survey (AHS).\(^{35}\)

The probability of mortgage market re-entry after default \( \varphi \) is 0.143. This matches the 7-year period for which a foreclosure flag remains on a consumer’s credit report.\(^{36}\) The statutory LTV limit \( \theta \) is 85%.

Distribution of age-1 households

To initialize the stationary distribution of households, I stratify a sample of households with heads between ages 23–27 from the 2016 SCF into \( N_z \) groups according to their incomes such that the shares of households in each group matches the invariant probability distribution of the income shock \( z \).\(^{37}\) For each income group, I calculate the homeownership rate, the fraction of homeowners with mortgages, mean liquid assets, mean home equity, and the mean interest rate on the first mortgage. Within each \( z \), I distribute households across states to match both the statistics computed from the SCF and the invariant probability distributions of the depreciation shock \( \delta \) and the house price shock \( p \).

5.2 Internally calibrated parameters

The remaining 15 parameters of the model are chosen by minimizing the weighted distance between an equal number of empirical targets and their equivalents in the stationary distribution of the model. The targets fall into three broad categories. I calculate moments relevant to mortgage market outcomes from the Freddie Mac Single Family Loan-Level Dataset.\(^{38}\) I calculate targets pertaining to cross-sectional and life-cycle features of household balance sheets from the 2016 SCF.\(^{39}\) Two moments, the mean foreclosure and housing

\(^{35}\)See Appendix B.2 for additional details. The average offset for borrowers in the stationary distribution of the calibrated model is 20%.

\(^{36}\)See https://www.experian.com/blogs/ask-experian/how-long-does-a-foreclosure-stay-on-your-credit-report/.

\(^{37}\)\( N_z \) is the number of discretized income states in the numerical solution of the model. I set \( N_z = 5 \). The quantitative results are robust to using more grid points for the income shock.

\(^{38}\)I calculate these moments using a sub-sample of mortgages originated in 2016 to ensure consistency with the 2016 SCF.

\(^{39}\)I exclude households in the top 1% of the net worth distribution when doing so. This is because the SCF is designed to over-sample households in the right tail of the wealth distribution for home equity

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depreciation rates, are from aggregate data sources.

Preferences and endowments

Working and retired households differ in their utility weight on nondurable consumption \( \alpha_j \). The calibration implies that working households place less weight on nondurable consumption relative to retired households (0.703 versus 0.728, respectively). This helps the model match the ratio of aggregate housing wealth to income. The subjective discount factor \( \beta \) is 0.911, similar to other values found in the literature, and is informative about the ratio of aggregate net worth to income. The weight on the bequest motive \( B \) is 27.825 and limits declines in net worth late in life. The flow utility loss from default \( \xi \) is 8.782.

Housing

Three parameters govern the distribution of owner-occupied house sizes: the minimum house size \( h \), the ratio between the maximum and minimum house sizes \( h_{\text{gap}} \), and the spacing parameter for the housing grid \( h_{\text{skew}} \). The minimum house size is set to 7.399, the largest house size is 1.343 times larger, and the spacing parameter is 0.977. This implies a slightly right-skewed distribution of house sizes. These parameters are pinned down by the 25th and 75th percentiles of the distribution of home equity relative to net worth.

The large depreciation shock \( \delta \) is set to 33.2%. Negative equity is a necessary but not sufficient condition for default in this class of models. Because the LTV constraint prevents households from beginning their tenure as owners with negative equity and house prices in the model are mean reverting, a large depreciation shock acts as a direct shock to expenditures that can make default optimal for some underwater homeowners. The probability of a large depreciation shock \( \zeta \) is chosen such that the expected value of the depreciation shock is a smaller share of their overall savings—see, e.g., Kuhn and Rios-Rull (2020). Appendix B.3 contains additional information on how these moments are defined and calculated.

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\(\text{Guren, Krishnamurthy and McQuade (2020)}\) make a similar assumption in their calibration.

\(\text{Compared to representative-agent models, a lower discount factor is typically needed in models with heterogeneous agents in order to balance out the precautionary savings motive. For instance, Berger et al. (2017) set} \ \beta = 0.918, \ \text{and Corbae and Quintin (2015) set} \ \beta = 0.876.\)

\(\text{Letting} \ N_h \ \text{be the number of housing grid points and} \ h = h_{\text{gap}} \ h, \ \text{the value of the} \ i^{\text{th}} \ \text{grid point is} \)

\[ h_i = \left( \frac{i - 1}{N_h - i} \right) h_{\text{skew}} (\bar{h} - \bar{h}) + h. \]

\(h_{\text{skew}} = 1\) results in a linearly spaced grid, and \(h_{\text{skew}} > 1\) produces a left skewed grid, and \(h_{\text{skew}} < 1\) yields a right skewed grid.

\(\text{For default to be optimal, a borrower must have both negative home equity and experience an adverse shock to their cash on hand. Foote, Girardi and Willen (2008), among many other papers, discuss the} \ \text{“double-trigger” hypothesis of foreclosure.}\)
2.27%. Its value implies that an owner is hit by a large depreciation shock once every 27 years. The housing adjustment cost $\kappa_h$ is 0.480, and the rent-price ratio $R$ is 2.430.

**Mortgages**

The default risk threshold for a high-DTI loan $\Psi$ is set to 0.025%. The cost of default on a low-DTI loan to lenders $\gamma_L$ is set equal to the calibrated value of the housing adjustment cost. This reflects the fact that, in reality, lenders are responsible for selling properties repossessed after a foreclosure. Because high-DTI loans are not subject to any additional legal liability relative to low-DTI loans in the event of default under current regulations for the conforming loan market, I set $\gamma_H = \gamma_L$.\(^{44}\)

The mortgage adjustment cost $\kappa_m$ is 0.321. Together with the housing adjustment cost, it implies that a household who finances a new house purchase with a mortgage pays around 9% of their house value in transaction costs.\(^{45}\) The mortgage servicing cost $\phi$ is 1.314% and pinned down by the spread between the risk-free rate and the average real mortgage interest rate at origination.

**5.3 Model fit**

Table 3 list the internally calibrated parameters, the empirical targets, and their corresponding model moments. Overall, the model does well at replicating the targeted moments, including the aggregate foreclosure and homeownership rates. Importantly, it matches the share of borrowers with DTI ratios above 45%, the share of borrowers with DTI ratios between 43% and 45%, and the mean DTI ratio at origination—key features of the empirical distribution of DTI ratios.

The share of mortgages with a DTI ratio above 45% is highly informative about the parameterization of the default risk threshold for a high-DTI loan $\Psi$, which is set to 0.025%. To place the strictness of this threshold in perspective, 59% of borrowers who obtain new loans in the stationary distribution of the model have an endogenous probability of default less than this value.\(^{46}\) The calibration implies that the pool of borrowers who are sufficiently

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\(^{44}\)The two foreclosure cost parameters, $\gamma_L$ and $\gamma_H$, are not directly involved in computing the calibration loss function but depend on the housing adjustment cost, which is.

\(^{45}\)This is somewhat larger than other estimates from the literature. Berger and Vavra (2015) estimate that the fraction of the value of durable goods lost to adjustment costs is 5.3%. Their model features only one durable asset, whereas my model has both illiquid housing and mortgage debt. The fixed cost of obtaining a new loan in Boar, Gorea and Midrigan (2020) is 2.3% of mean per-capita income, but their model also features an i.i.d. utility loss from refinancing that represents a non-pecuniary adjustment cost. Keys, Pope and Pope (2016) document that many homeowners do not refinance when theory predicts they should, so large transaction costs are needed in the model to rationalize observed patterns of loan adjustment.

\(^{46}\)In the data, 60% borrowers have a credit score of at least 740, which, as discussed in Section 3.3, is the
Table 3. Internally calibrated parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{j &lt; T_h}$ Working HH nondur. cons. weight</td>
<td>0.703</td>
<td>Aggregate housing stock</td>
<td>2.57</td>
<td>2.65</td>
</tr>
<tr>
<td>$\alpha_{j \geq T_h}$ Retired HH nondur. cons. weight</td>
<td>0.728</td>
<td>DTI ratio at origination</td>
<td>0.35</td>
<td>0.38</td>
</tr>
<tr>
<td>$B$ Bequest weight</td>
<td>27.825</td>
<td>Ret. to working HH net worth</td>
<td>1.94</td>
<td>1.92</td>
</tr>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.911</td>
<td>Aggregate net worth</td>
<td>2.64</td>
<td>2.64</td>
</tr>
<tr>
<td>$\delta$ High depreciation rate</td>
<td>0.332</td>
<td>Mean LTV ratio</td>
<td>0.37</td>
<td>0.35</td>
</tr>
<tr>
<td>$h_{gap}/\bar{h}$ Small house size</td>
<td>1.343</td>
<td>75th pctile. home eq. share</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>$h_{skew}$ Housing grid skewness</td>
<td>0.977</td>
<td>% loans with DTI ∈ (43, 45]</td>
<td>10.24</td>
<td>10.22</td>
</tr>
<tr>
<td>$h$ Smallest house size</td>
<td>7.399</td>
<td>25th pctile. home eq. share</td>
<td>0.71</td>
<td>0.56</td>
</tr>
<tr>
<td>$\kappa_h$ Housing transaction cost</td>
<td>0.480</td>
<td>Homeownership rate</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$\kappa_m$ Mortgage transaction cost</td>
<td>0.321</td>
<td>Share of owners with debt</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>$\phi$ Mortgage servicing cost (%)</td>
<td>1.314</td>
<td>Mean mort. interest rate (%)</td>
<td>2.62</td>
<td>2.61</td>
</tr>
<tr>
<td>$\Psi$ High DTI def. prob. threshold (%)</td>
<td>0.025</td>
<td>% loans with DTI &gt; 45</td>
<td>8.71</td>
<td>8.59</td>
</tr>
<tr>
<td>$R$ Rent-price ratio</td>
<td>2.430</td>
<td>Aggregate liquid wealth</td>
<td>0.98</td>
<td>0.84</td>
</tr>
<tr>
<td>$\xi$ Utility loss from def.</td>
<td>8.782</td>
<td>Foreclosure rate (%)</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>$\zeta$ High depreciation shock prob. (%)</td>
<td>3.770</td>
<td>Mean depreciation rate (%)</td>
<td>2.27</td>
<td>2.27</td>
</tr>
<tr>
<td>$\gamma_L$ Low-DTI loan def. cost</td>
<td>0.480</td>
<td>Set to housing adjustment cost $\kappa_h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_H$ High-DTI loan def. cost</td>
<td>0.480</td>
<td>Set to $\gamma_L$ in baseline calibration</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The values for aggregate housing stock, net worth, and liquid wealth are scaled by mean household income in the stationary distribution. LTV ratio is mortgage debt relative to home value in the cross section, not at the time of loan origination. Home equity share is the fraction of net worth held in the form of home equity. $\bar{h}$ denotes the largest house size in the model.

creditworthy to benefit from lender discretion is fairly large but that the default risk threshold nevertheless binds for a nontrivial share of households.

The minimum house size is important for matching the observed share of borrowers bunched at the DTI limit. Intuitively, increasing the smallest house size available to agents also increases loan balances and DTI ratios at origination. Due to borrower selection effects, however, this only occurs to up a point. If the minimum house size becomes too large, then some homeowners will instead find it optimal to rent. The remaining population of homeowners has higher incomes and net worth such that, in equilibrium, loan balances and DTI ratios at origination decline.

The relatively lower weight on housing services in the utility function of retired households helps the model match the mean DTI ratio at origination. The assumptions that mortgage maturity is decreasing in age and that all debt must be repaid at the end of life can mechanically generate very high DTI ratios.\textsuperscript{47} Lowering retired households’ utility weight

\textsuperscript{47}All else equal, retired households with a relatively high preference weight on housing services take out larger loans, which are then amortized over a relatively short remaining lifetime. Retired households also have lower incomes. Together, these generate large DTI ratios. This tendency of the model can be modified by making loan maturity independent of age, but doing so requires tracking another state variable.
Figure 5. Household consumption, income, and balance sheets over the life cycle

Notes: Nondurable consumption and income are both normalized relative to households with heads between the ages 25–32. LTV ratio is outstanding mortgage debt scaled by value of the primary residence. Debtor share is the fraction of homeowners with positive mortgage debt. Data source: 2016 SCF for household income and net worth, 2017 PSID for nondurable expenditures.

on housing services keeps the model from generating counterfactually large DTI ratios.

The model captures important cross-sectional and life-cycle dimensions of household income, consumption, and savings. Figure 5 shows that the model produces the familiar hump-shaped profiles of nondurable consumption and income. Parameters governing the income process in the model are directly assigned, so it is unsurprisingly that the model reproduces mean household income by age.\footnote{Income life-cycle profiles in the data and model do not match exactly because I use the SCF in Figure 5 but the PSID to parameterize the income process.} I do not, though, target any moments related to consumption. The same figure shows the evolution of household net worth and its composition as households age. With the exception of the net worth ratio of retired to working households, life-cycle patterns of household wealth are not targeted in
the calibration. The model matches well the increase in net worth as households age and its breakdown into liquid assets and home equity, along with the gradual decrease in leverage. It is also broadly consistent with homeownership rates and the extensive margin of debt among owners over the life cycle. In the model, homeownership flattens out in retirement whereas it continues to rise in the data. This stems from a tension in the calibration discussed above whereby a lower weight on housing services in the utility function of retired agents is needed to match the average DTI ratio at origination. Quantitatively, the trade-off is a slight counterfactual decrease in homeownership during retirement relative to working age.

6 Quantitative results of the model

Having validated that the calibrated model matches the empirical distribution of borrowers’ debt payment-to-income ratios and the consumption and savings behavior of households over the life cycle, I now establish that it successfully captures the novel stylized facts documented in Section 3. The model quantitatively replicates the discontinuity in default risk observed at the 45% debt payment-to-income limit. I show that the endogenous contract choice and screening technology are needed to explain this: a standard model without these features counterfactually predicts that default risk is strictly increasing in the DTI ratio and overstates the dispersion of default risk across borrowers.

The calibrated model reveals why an identical DTI limit binds for households in different regions of the state space. I show that whether a household is constrained by the limit along the intensive or extensive margin of mortgage debt is informative about this heterogeneity. Agents constrained along the extensive margin are current renters transitioning to homeownership who exhibit the highest propensity to default. Their exit from the mortgage market in response to the DTI requirement reduces the overall variance in default risk among borrowers obtaining new loans. By contrast, agents constrained along the intensive margin are existing homeowners who refinance in order to relax current and future liquidity constraints. Their high level of home equity makes default unlikely for these households. Their switch from a high- to low-DTI loan accounts for the sharp discontinuity in default risk observed at the current 45% cutoff. Both groups of households have high desired DTI ratios but vary substantially in their ability to repay, indicating that the DTI ratio itself is an insufficient proxy for creditworthiness.

6.1 Borrower selection into high-DTI loans

The quantitative success of the calibrated model rests on its ability to yield the observed difference in default risk of borrowers on either side of the statutory debt payment-to-
The model does well in this regard, generating a discontinuity in interest rates at the statutory DTI limit in line with the empirical estimates from Section 3.2. This can be seen in the left-hand panel of Figure 6, which plots mortgage interest rates as a function of DTI ratios at origination in the stationary distribution of the model and in the loan-level data. Because the model is parameterized to match the residential mortgage market under current underwriting standards, the relevant empirical benchmark is mortgages originated since 2009 (i.e., the “post-DTI limit” line in Figure 2). In the model, the interest rates paid by borrowers with DTI ratios between 43% and 45% is 9.4 basis points greater than those above the limit, compared to 6.7 in the data. The equilibrium cost of borrowing associated with high-DTI loans is 1.5 basis points below average, compared to 1.4 in the data. Therefore, the model is quantitatively consistent with the non-monotonic relationship between default risk and DTI ratios documented in the data.

The endogenous loan choice and the screening technology in the model are essential for
explaining this fact. The right-hand panel of Figure 6 plots the equilibrium relationship between interest rates and DTI ratios in a version of the model in which the default risk threshold for high-DTI loans is entirely relaxed and all other parameters in the model are unchanged. This collapses my model to the standard theoretical framework in which households face only a loan-to-value constraint when obtaining a new loan.\textsuperscript{49} The relevant empirical benchmark in this case is mortgages originated prior to the introduction of the current DTI limit in 2009.

The standard framework is unable to generate non-monotonicity in the relationship between borrowers’ default risk and DTI ratios. Instead, default risk is strictly increasing in the DTI ratio at origination, qualitatively replicating the correlation that existed when mortgage underwriting standards did not include a binding DTI limit. This version of the model over-predicts the variance of default risk associated with mortgage originations: relative to the data, too many households with a high propensity to default in their current state enter into new loans. I speculate that, in reality, mortgage lenders performed some informal screening of their loan applicants that limited the overall riskiness of originations even in the absence of a formal DTI requirement.\textsuperscript{50} This suggests that the standard framework requires some additional friction that curtails the entry of least creditworthy households into the mortgage market.

In Appendix B.4, I present additional quantitative results on the selection of relatively creditworthy borrowers into high-DTI loans. These show that, under the current DTI limit, the \textit{ex ante} default probability of borrowers in the model is increasing in the DTI ratio at origination to the left of the limit and discontinuous at the 45% cutoff. This is similar to the relationship between credit scores and DTI ratios at origination documented in the data.

### 6.2 Identifying the characteristics of constrained borrowers

Next, I identify the characteristics of borrowers who are constrained by the current debt payment-to-income limit and their responses to it. This rationalizes the observed non-linear relationship between default risk and DTI ratios and highlights the potential for welfare-improving policy reform. The advantage of using a structural model is that it allows me

\textsuperscript{49}The intuition for this begins with the fact that, because the foreclosure costs of a low- and high-DTI loan are identical, the lender’s zero-profit condition used to price loans is independent of contract type. Removing the qualifying default risk threshold for high-DTI loans is accomplished by setting $\Psi = 1$. In words, a household who optimally demands a loan with a DTI ratio above the statutory limit can obtain one regardless of their creditworthiness, as long as the loan can be supported in equilibrium. This means that loans with a DTI ratio above 45% are screened no more than loans with a DTI ratio less than or equal to 45%. Therefore, there is no meaningful distinction between low- and high-DTI loans in this counterfactual parameterization of the model.

\textsuperscript{50}To use one example, Freddie Mac’s selling guidelines have long required additional justification from a lender when originating a mortgage with a DTI ratio above 36%.
to compute the counterfactual loan choices of households, including those who would not appear in data on mortgage originations. As a result, a constrained household is defined as an agent who would optimally chose a high-DTI loan but has an endogenous default risk above the creditworthiness threshold that makes this option infeasible.

I show that the current DTI limit binds not just for borrowers observed at the 45% cutoff and that adjustments along both the intensive and extensive margins of debt are needed to quantitatively account for the discontinuity in default risk at the DTI limit from Figure 6. The extensive margin accounts for 80% of all constrained households in the model: after the introduction of the DTI limit, they switch from a high-DTI loan to not obtaining a mortgage at all. These households constitute the riskiest subset of borrowers in a no-DTI limit setting, and their exit from the mortgage market lowers the dispersion in default risk among remaining borrowers. The other 20% of constrained households adjust along the intensive margin of debt by switching from a high- to a low-DTI loan, decreasing their loan size until their DTI ratio falls below the statutory limit. The shift of these borrowers within the distribution of DTI ratios accounts for the difference in default risk around the limit.

The importance of adjustments along the extensive margin of mortgage debt is corroborated by empirical evidence from DeFusco, Johnson and Mondragon (2020). They document that, in response to the introduction of a DTI limit in the jumbo loan market, the disappearance of high-DTI loans that would have been originated in the absence of the policy is larger than what existing estimates of the elasticity of mortgage demand to interest rates would imply. They argue that this reflects changes in credit supply and the exit of lenders from the affected segment of the market. The screening technology in my model generates a similar outcome whereby loans that are deemed too risky are simply not originated.

To better understand the trade-offs involved in the optimal choices of constrained households, it is helpful to first study the characteristics of high-DTI borrowers in the stationary distribution of the model. In the top half of Table 4, I compare the balance sheets and income of high-DTI borrowers and constrained households, who are further split based on whether they are constrained on the intensive or extensive margin.51 Virtually all high-DTI borrowers are current homeowners, indicating that their loans are refinances. Perhaps unsurprisingly, they have low current incomes, which mechanically tightens the DTI constraint and raises the value of obtaining a high-DTI loan. The high-DTI option is feasible for them not merely because they have relatively high net worth but also because,

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51 A negligible fraction of constrained borrowers—around 0.4%—can be found among households who obtain new loans with DTI ratios below 43%. Around 7% of debtors who default are also constrained. Because the total share of defaulting households is already small, these households account for only 0.2% of all constrained households. No constrained households are found among owners who continue with an existing loan or households who adjust their house size without simultaneously obtaining a mortgage.
Table 4. Characteristics of constrained households in the model

<table>
<thead>
<tr>
<th></th>
<th>DTI &gt; 45</th>
<th>DTI ∈ (43,45]</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Const.</td>
<td>All</td>
</tr>
<tr>
<td>Income</td>
<td>1.17</td>
<td>1.22</td>
<td>1.43</td>
</tr>
<tr>
<td>Liquid wealth</td>
<td>2.41</td>
<td>1.06</td>
<td>1.26</td>
</tr>
<tr>
<td>Home equity</td>
<td>3.03</td>
<td>1.03</td>
<td>1.21</td>
</tr>
<tr>
<td>Net worth</td>
<td>5.45</td>
<td>2.10</td>
<td>2.47</td>
</tr>
<tr>
<td>Owner share</td>
<td>0.92</td>
<td>0.76</td>
<td>0.80</td>
</tr>
<tr>
<td>Min. loan payment</td>
<td>0.43</td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>Maintenance costs</td>
<td>0.29</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>Income shock</td>
<td>0.74</td>
<td>0.65</td>
<td>0.80</td>
</tr>
<tr>
<td>House price shock</td>
<td>1.14</td>
<td>0.95</td>
<td>1.07</td>
</tr>
</tbody>
</table>

|                         | Low-DTI loan |     |     |     |     |
| Loan size               | 3.13         | 5.34| 6.25| 2.96| 2.42|
| Int. rate (%)           | 3.14         | 2.72| 2.69| 5.16| 5.37|
| Def. prob. (%)          | 22.87        | 2.08| 1.83| 89.30| 94.26|
| Liq. savings            | 0.58         | 0.68| 1.15| 0.02| 0.01|
| Consumption             | 0.34         | 0.43| 0.59| -0.05| -0.01|

|                         | High-DTI loan |     |     |     |     |
| Loan size               | 5.75         | 6.05| 6.59| 7.06| 7.11|
| Int. rate (%)           | 2.60         | 2.75| 2.71| 3.13| 4.85|
| Def. prob. (%)          | 0.00         | 2.47| 2.03| 8.04| 75.01|
| Liq. savings            | 2.85         | 1.36| 1.47| 1.28| 0.30|
| Consumption             | 0.59         | 0.49| 0.62| 0.35| 0.07|
| % of households         | 2.92         | 1.69| 3.53| 6.94| 31.04|

Notes: The first row sorts households according to their loan choices in the stationary distribution of the model. The “constrained” column of borrowers who choose loans with DTI ratios between 43% and 45% are households constrained along the intensive margin by the DTI limit. The “constrained” column of borrowers who rent are households constrained along the extensive margin by the DTI limit. Income, balance sheet, and expenditure variables are in units of the numeraire. Owner share is to the fraction of households in a given group who are existing homeowners. The bottom half of the table displays optimal choices of households conditional on their loan contract choice in the stationary distribution of the model. Percent of households refers to the group’s share in the total population of the model.

as existing owners, they have significant positive equity stored in their homes.\textsuperscript{52} For any empirically plausible parameterization of housing and loan adjustment costs in the model, such an owner will never default because they can always exercise the option to sell instead, thereby converting illiquid home equity into cash on hand.

\textsuperscript{52}Aggregate net worth relative to income for high-DTI borrowers is 4.66, compared to 2.64 for all households.
Despite their high net worth, a high-DTI loan is optimal for these households because they value the additional liquidity attained through refinancing for precautionary reasons. These agents face a future stream of large mortgage payments on their existing loans; incur high maintenance costs that result from both positive depreciation and house price shocks; and experience a negative persistent income shock. The bottom half of Table 4 makes clear that, compared to their decisions conditional on a low-DTI loan, these households increase their liquid savings considerably under a high-DTI loan. Furthermore, because high-DTI borrowers have large housing outlays relative to cash on hand, a smaller mortgage is in some cases infeasible and would result in negative consumption in some future states of the world after all other expenses are met.

Households constrained along the intensive margin by the DTI limit have similar incomes to high-DTI borrowers but lower net worth, liquid wealth, and home equity. These constrained agents face smaller streams of required housing outlays, however. Because their future expected liquidity constraints are less binding, they can optimally substitute into low-DTI loans with slightly smaller balances at origination. These households account for around half of all borrowers found at the statutory limit. Because borrowers bunched below the limit are a mix of constrained households and agents for whom a DTI ratio near 45% was always optimal, the model can match the meaningful heterogeneity in default risk observed at the DTI limit.

Households constrained along the extensive margin differ significantly in their underlying characteristics compared to high-DTI borrowers. Almost 90% of them are existing renters, indicating that the high-DTI loans they would have obtained are purchase loans that would have allowed them to become owners. Corroborating this is the fact that, compared to all renters in the model, they have a much higher stock of liquid assets that would be needed to cover the down payment and associated adjustment costs for a new house. The indivisibility of owner-occupied housing, captured by a minimum house size in the model, is crucial for this result: it introduces lumpy adjustment to housing by effectively bounding the face value of a purchase loan from below. A smaller loan that takes the form of the low-DTI contract is therefore infeasible for agents who need a mortgage for this reason, as shown

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53 Some of this reflects the fact that 24% of borrowers at the DTI limit are current renters for whom the mortgage is a purchase loan. Existing renters, by definition, have zero home equity. Conditional on being an existing owner, mean home equity of borrowers at the limit is 1.48 units of the numeraire. This is still much lower than the mean home equity of high-DTI borrowers of 3.03.

54 In Appendix B.5, I provide additional results for the heterogeneity of default risk at the DTI limit in the model.

55 Purchase loans are associated with larger adjustment costs in absolute terms than refinances because the borrower is changing their house size in addition to getting a new loan. These transaction costs are also proportionally more costly for households with low cash on hand.
by the negative consumption it implies. As a result, these households find it optimal to forgo homeownership altogether and continue renting. Among all debtors, new homeowners transitioning from renting exhibit the highest likelihood of default because they begin their tenure as owners with low equity in their homes. These leaves them vulnerable to adverse future shocks that can make default optimal.

7 The equilibrium effect of DTI limits: the case of Dodd-Frank

Having established that the model generates heterogeneity in default risk across borrowers conditional on their debt payment-to-income ratios, I use it to evaluate the aggregate and distributional implications of a proposed reform to DTI limits from the Dodd-Frank Act. This reform lowers the statutory limit from 45% to 43% and increases the costliness of high-DTI loans to lenders while granting them more discretion to originate them. Policymakers and market participants have disagreed significantly over its potential effect on the quantity and price of mortgage credit, particularly in light of DTI ratios rising in recent years.\textsuperscript{56}

From the borrower’s perspective, the main trade-off is an expanded feasible set of high-DTI mortgages—facilitated by greater lender discretion—versus a higher equilibrium interest rate on those loans. The tighter DTI constraint adds a further complication, given that 10% of existing mortgage originations satisfy the current limit but not the proposed one. The model is uniquely suitable for evaluating this policy because it allows the incentives and discretion of lenders to comply with underwriting standards to vary meaningfully with the borrower’s endogenous loan contract choice. In a framework without this feature, this reform could only be modeled as the tightening of a strict DTI limit. This would omit potential welfare gains that accompany greater lender discretion.

The model predicts that this reform improves aggregate welfare by 1.31% in consumption-equivalent terms. Welfare gains from more lender discretion outweigh welfare losses from a rise in borrowing costs. Renters constrained along the extensive margin by the current DTI limit who qualify for a high-DTI loan under the more relaxed default risk threshold benefit most. Welfare losses are concentrated among households whose current low-DTI loans become high-DTI loans under the new policy. The higher their default risk, the more their equilibrium interest rates reflect the increased cost of originating high-DTI loans.

7.1 Description of the Dodd-Frank ability-to-repay rule

I study the ability-to-repay rule from the Dodd-Frank Act of 2010, which states:

\textsuperscript{56}See Moskins (2014). The reform came into effect for jumbo loans in 2014 but its implementation in the conforming loan market has been delayed.
Table 5. Mortgage contract parameters under the current and new policies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Current</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_L$</td>
<td>Default cost of low-DTI loan</td>
<td>0.48</td>
<td>0.48</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>Default cost of high-DTI loan</td>
<td>0.48</td>
<td>2.15</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Statutory DTI limit</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Default risk threshold for high-DTI loan (%)</td>
<td>0.03</td>
<td>3.55</td>
</tr>
</tbody>
</table>

Notes: The default cost parameters are in units of the numeraire.

“...no creditor may make a residential mortgage loan unless the creditor makes a reasonable and good faith determination based on verified and documented information that, at the time the loan is consummated, the consumer has a reasonable ability to repay the loan.”

The rule creates an incentive for lenders to originate low-DTI loans without banning high-DTI loans outright.\(^57\) A back-end DTI ratio of 43% or lower is a sufficient condition for lenders to satisfy this rule.\(^58\) A lender who originates a mortgage with a larger DTI ratio, though, is subject to greater legal liability if the borrower eventually defaults on the loan.

Table 5 lists the changes to parameter values that implement the Dodd-Frank reform in the model. The statutory DTI limit $\lambda$ is lowered from 45% to 43%. To capture the greater legal liability assumed by a lender who originates a high-DTI loan, I increase its cost of default to the lender such that $\gamma_H > \gamma_L$. I calibrate $\gamma_H$ by following a cost-benefit analysis of the reform conducted by the Consumer Financial Protection Bureau (2013).\(^59\) This implies that the resource loss resulting from default on a high-DTI loan is 4.5 greater than that on a low-DTI loan. Finally, to discipline the degree of lender discretion associated with the rule, I set the default risk threshold for a high-DTI loan $\Psi$ to 3.55% such that, in the stationary distribution of the model under the reform, around one-fourth of borrowers choose a high-DTI loan. This matches the observed share of mortgage originations in 2019 that had a DTI ratio above 43%.\(^60\) All other parameter values remain unchanged.

\(^57\)The full text of the rule is found in Sections 1411 and 1422 of Title XIV of the Dodd-Frank Act. See https://www.congress.gov/bill/111th-congress/house-bill/4173/text.

\(^58\)The 43% DTI cap is not in the original legislation but, rather, the Consumer Financial Protection Bureau’s (CFPB) implementation of the rule. The CFPB additionally requires mortgages satisfy a ban on certain loan features deemed too risky—such as negative amortization and interest-only payments—and a cap on points and fees due at origination in order to comply with the ability-to-repay rule. I abstract from those features in my policy counterfactual.

\(^59\)See Appendix B.6 for details about this parameterization, where I also demonstrate that my welfare results are robust to different values of $\gamma_H$.

Table 6. Aggregate effects of the Dodd-Frank reform

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default rate (%)</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>Homeownership rate</td>
<td>0.68</td>
<td>0.71</td>
</tr>
<tr>
<td>Share of owners with mortgage</td>
<td>0.63</td>
<td>0.69</td>
</tr>
<tr>
<td>LTV ratio</td>
<td>0.63</td>
<td>0.61</td>
</tr>
<tr>
<td>DTI ratio</td>
<td>0.38</td>
<td>0.44</td>
</tr>
<tr>
<td>Mortgage interest rate (%)</td>
<td>2.61</td>
<td>2.67</td>
</tr>
<tr>
<td>Share of high-DTI mortgages</td>
<td>0.09</td>
<td>0.25</td>
</tr>
<tr>
<td>Share of mortgages with DTI &gt; 43%</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>Aggregate net worth</td>
<td>2.69</td>
<td>2.65</td>
</tr>
<tr>
<td>Aggregate liquid wealth</td>
<td>0.90</td>
<td>0.85</td>
</tr>
<tr>
<td>Aggregate home equity</td>
<td>1.79</td>
<td>1.80</td>
</tr>
</tbody>
</table>

Notes: Moments are calculated from the respective stationary distributions of the model under the two policies. LTV ratio, DTI ratio, and mortgage interest rate at reported at origination. Aggregate net worth, liquid wealth, and home equity are scaled relative to income.

7.2 Aggregate effects of the Dodd-Frank ability-to-repay rule

Table 6 presents aggregate outcomes in the steady state of the model under the current and proposed DTI policies. Despite a tighter DTI constraint, the average default rate rises from 0.7% to 0.8% under the Dodd-Frank ability-to-repay rule. This is because, due to the greater ease of obtaining high-DTI loans, more default-prone households become homeowners. This is shown by the increases in the mortgage interest rate, the share of high-DTI loans, and the fraction of households who own houses and have outstanding debt.

Taken together, the aggregate results underscore a broader trade-off that policymakers face between limiting default and ensuring access to homeownership. The DTI limit is most likely to bind for low-income and/or low-wealth households who endogenously demand larger loans in order to switch from renting to owning or to forestall liquidating their current houses. Such households also exhibit a higher propensity to default in equilibrium. Relaxing the default risk threshold for high-DTI loans necessarily broadens the composition of borrowers and owners to include more of these households.

The increase in homeownership that results from lenders having more discretion to originate high-DTI loans accounts for the aggregate welfare gain in the stationary economy under the reform. In consumption-equivalent terms, the average welfare change is 1.31%.\(^{61}\)

In a setting with incomplete markets, housing wealth and the ability to smooth consumption

---

\(^{61}\)I define consumption-equivalent welfare change in Appendix B.7. The welfare changes in this paper abstract from transition dynamics and represent a comparison of one stationary economy to another.
Table 7. Welfare changes by housing tenure transitions

<table>
<thead>
<tr>
<th></th>
<th>Share (%)</th>
<th>CEV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own-to-own</td>
<td>68.19</td>
<td>0.78</td>
</tr>
<tr>
<td>Own-to-rent</td>
<td>0.47</td>
<td>−12.67</td>
</tr>
<tr>
<td>Rent-to-own</td>
<td>2.88</td>
<td>29.24</td>
</tr>
<tr>
<td>Rent-to-rent</td>
<td>28.47</td>
<td>−0.01</td>
</tr>
</tbody>
</table>

Notes: “Own” and “rent” refer to a household’s optimal housing tenure choice under a given policy, not their predetermined housing wealth.

Table 7 illustrates this by displaying the consumption-equivalent welfare changes for households conditional on changes in housing tenure. The highest welfare gains accrue to households in the model who switch from renting to owning—intuitively, these are households who qualify for a high-DTI loan under the reform but not the current policy—while the highest welfare losses are experienced by households who make the opposite switch. The welfare effect on households who rent under both policies is negative but minimal. Changes in credit conditions only affect these households through their continuation values, and most households who are renters in equilibrium have low probabilities of becoming owners in the future due to the persistence of income and house price shocks. Households who choose to own under both policies experience a small but positive welfare gain. For these agents, a less strict default risk threshold for high-DTI loans improves their ability to smooth consumption and relaxes their current and future liquidity constraints in states of the world when they are most likely to bind.

To demonstrate the quantitative importance of lender discretion in affecting the size of these changes in housing tenure, I consider alternative parameterizations of the default risk threshold for high-DTI loans $\Psi$ in Appendix B.8. The aggregate homeownership rate and welfare change are both increasing in the laxness of the default risk threshold. It also highlights that the welfare implications of reforms to DTI limits are sensitive to the degree of lender discretion.

It should be noted that, in my theoretical environment, I have abstracted from negative through mortgage debt are valuable from a self-insurance perspective. This is true even though home equity is an illiquid store of wealth.\textsuperscript{62} In exchange, households are willing to tolerate a rise in their likelihood of future default and an increase in their cost of borrowing.

\textsuperscript{62}In an earlier version of their paper, Boar, Gorea and Midrigan (2020) show that consumption smoothing in a model with two assets, one liquid and one illiquid, is higher than an otherwise identical economy with a single liquid asset. See Hurst and Stafford (2004) for empirical evidence on home equity extraction as a mechanism for consumption insurance.
externalities associated with default and leverage. My calculations should be thought of as an upper bound on the welfare effects of the Dodd-Frank reform. I have also abstracted from equilibrium house prices in my model. As Kaplan, Mitman and Violante (2020) have shown, the assumptions of long-term mortgage contracts and a rental housing market with complete integration together imply that changes in credit conditions exhibit little pass-through to equilibrium house prices. Because my model shares these two features, incorporating feedback effects through house prices should not change my results substantially.

7.3 Distributional effects of the Dodd-Frank ability-to-repay rule

Relative to the status quo, the key trade-off that confronts households under the reform is (1) an expanded feasible set made possible by a more relaxed qualifying threshold for a high-DTI loan versus (2) a higher cost of borrowing potentially implied by a decrease in the lender’s payoff from default. The largest first order effects of this reform will be felt by households who are constrained by current regulations. I show that households currently constrained along the extensive margin but creditworthy enough to qualify for a high-DTI loan and become homeowners under the reform benefit most. The effects on households constrained along the intensive margin are ambiguous because their current low-DTI loan becomes a high-DTI loan under the reform. These agents are most affected by the increase in the implicit cost of high-DTI loans. The extent to which they continue with their existing contract choice depends on the degree to which lenders pass on this higher cost to borrowers. That, in turn, hinges on their equilibrium default risk.

First, I consider the welfare changes and outcomes for households currently constrained along the extensive margin of debt. I show in the last column of Table 8 that 41% of these households are able to obtain a high-DTI loan and become homeowners under the reform.63 Although the average equilibrium interest rate on these mortgages is high at 3.15%, the benefits from this change in housing tenure outweigh this and account for the large 12.4% increase in welfare. 58% of these households remain too risky to qualify for a high-DTI loan and continue renting. However, as shown in Section 7.2, their welfare is relatively unaffected.

Next, to clarify the reform’s ambiguous effects on the welfare of households currently constrained along the intensive margin of debt, I split them into “safe” and “risky” subsets.64 Obtaining what becomes high-DTI loan is infeasible for 20% of the risky subset because they do not meet the default risk threshold now associated with their choice of loan contract. Reducing their loan size by becoming low-DTI borrowers is also infeasible, using

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63A minuscule fraction of them, around 0.02%, choose to own and obtain low-DTI loans.
64A household is safe or risky based on whether their probability of default under the current policy is less or more than 0.025%, respectively. In the baseline calibration, 0.025% is the maximum default probability a borrower can exhibit and still qualify for a high-DTI loan under current regulations.
Table 8. Effect of Dodd-Frank reform on households constrained by the current DTI limit

<table>
<thead>
<tr>
<th></th>
<th>DTI ∈ (43, 45)</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Safe</td>
</tr>
<tr>
<td>CEV (%)</td>
<td>0.21</td>
<td>5.39</td>
</tr>
<tr>
<td>Share with CEV ≥ 0</td>
<td>0.79</td>
<td>1.00</td>
</tr>
<tr>
<td>Share who rent</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td>Share who get low-DTI loan</td>
<td>0.06</td>
<td>0.32</td>
</tr>
<tr>
<td>Share who get high-DTI loan</td>
<td>0.76</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Low-DTI loan

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Safe</th>
<th>Risky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan size</td>
<td>5.33</td>
<td>3.30</td>
<td>6.75</td>
</tr>
<tr>
<td>Int. rate (%)</td>
<td>2.70</td>
<td>2.63</td>
<td>2.74</td>
</tr>
<tr>
<td>Def. prob (%)</td>
<td>2.12</td>
<td>1.21</td>
<td>2.75</td>
</tr>
<tr>
<td>Liq. savings</td>
<td>0.46</td>
<td>0.78</td>
<td>0.23</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.42</td>
<td>0.46</td>
<td>0.39</td>
</tr>
</tbody>
</table>

High-DTI loan

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Safe</th>
<th>Risky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan size</td>
<td>5.69</td>
<td>6.22</td>
<td>5.65</td>
</tr>
<tr>
<td>Int. rate (%)</td>
<td>2.94</td>
<td>2.66</td>
<td>2.96</td>
</tr>
<tr>
<td>Def. prob (%)</td>
<td>2.33</td>
<td>0.92</td>
<td>2.43</td>
</tr>
<tr>
<td>Liq. savings</td>
<td>1.56</td>
<td>2.72</td>
<td>1.47</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.46</td>
<td>0.50</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Notes: The first row denotes the choice of constrained households under the current policy. “Safe” and “risky” refer to whether borrowers had an expected probability of default less than or greater the qualifying default risk under the current policy, not the Dodd-Frank reform. “CEV” is the average welfare change in consumption-equivalent terms for a given subset of households. The bottom half of the table displays optimal choices of households conditional on their loan contract choice in the stationary distribution of the model under the Dodd-Frank reform. Loan size, liquid savings, and consumption are in units of the numeraire.

the argument from Section 6.2. Consequently, they become renters under the reform and experience a steep welfare loss. The remainder of the risky subset are sufficiently creditworthy that they can take advantage of increased lender discretion and switch to high-DTI loans. Compared to their choices under the current policy, they obtain slightly larger loans (5.65 versus 5.43) but at a higher interest rate (2.96% versus 2.72%) under the reform. This mutes their welfare gain. By contrast, the safe subset of households constrained along the intensive margin are uniformly better off under the Dodd-Frank reform. They can choose the high-DTI contract without a large increase in their equilibrium cost of borrowing. On average, the balance on a high-DTI mortgage chosen by this group is almost twice that on a low-DTI loan while the associated interest rate is only 3 basis points higher.

Figure 7 summarizes the distributional effects of the Dodd-Frank reform by plotting
Figure 7. Welfare changes by income and net worth quintiles under the Dodd-Frank reform

Notes: Cutoffs for income and net worth quintiles are calculated from the stationary distribution of households under the current policy. Blue and orange indicate a positive and negative welfare change, respectively.

welfare changes by income and net worth quintiles. Households with low incomes but medium levels of net worth correspond to liquidity constrained homeowners who can more easily qualify for high-DTI mortgages and use them to smooth consumption à la the wealthy hand-to-mouth households in Kaplan and Violante (2014). They also contain renters with large liquid savings who need large mortgages to become homeowners. Households with the highest incomes and net worth experience small changes to their welfare because they can already self-insure well against negative shocks. Welfare losses are concentrated among households with the lowest net worth but medium levels of income. They consist of households who switch from owning to renting, along with continuing renters whose probability of becoming owners has fallen due to tighter borrowing limits.

\[\text{Note: Because income and net worth are not independently distributed variables in this model, the measures of households in each income-net worth group are unequal.}\]
8 Conclusion

In this paper, I studied the role of borrower selection in the presence of a debt payment-to-income limit that binds for some, but not all, households in the mortgage market. I documented empirically that lenders have discretion to originate loans unconstrained by this requirement to relatively creditworthy borrowers. As a result, households with high desired DTI ratios are heterogeneous in their default risk. Introducing an endogenous loan contract choice and a simple screening technology to an otherwise standard incomplete-markets life-cycle model with a competitive market uniquely explains this fact. The calibrate model shows that a proposed reform to DTI limits that gives lenders more discretion to relax the DTI requirement for borrowers yields substantial welfare gains. This indicates that, in settings where home equity is valued as a form of self-insurance, there is merit in incorporating greater flexibility when designing household leverage regulations.

Looking ahead, there are a number of ways in which the questions pursued in this paper could be explored further. Incorporating endogenous house prices or an explicit externality arising from household leverage would permit an analysis of how effective DTI limits would have been at preventing foreclosures had they been in place during the Great Recession. Shocks to nominal interest rates are an important driver of mortgage refinancing decisions, which are themselves a channel for the transmission of monetary policy. An environment in which the risk-free rate is determined endogenously would be more suitable for studying business cycles. Finally, the model developed in this paper could be used to study the design of optimal limits on debt payment-to-income ratios.
References


A Empirical appendix

A.1 Additional figures

As context, Table 9 contains summary statistics for the sample of loans for the empirical analysis in Section 3. Because there is substantial time-series variation in these loan characteristics, Figure 8 plots the mean DTI ratio, high-DTI share, credit score, and interest rate by year. DTI ratios and the share of mortgages with DTI ratios above 45% fell during the Great Recession before recovering somewhat in recent years. Borrowers in the sample are on average more creditworthy now than they were prior to the Great Recession. Mortgage interest rates have trended downward since the mid-2000s.

Figure 9 displays the evolution of the distribution of DTI ratios from 2005 to 2016. There is little visual indication of a binding DTI constraint at any level during the housing boom. Since 2009, the share of mortgages with DTI ratios above 45% has decreased but constitute around 9% of all originations purchased by Freddie Mac. The share of mortgages with DTI ratios at or just below the statutory limit has increased over time. As of 2016, around 10% of mortgages have a DTI ratio between 43% and 45%.

In Figure 10, I show that the distribution of DTI ratios across mortgages purchased by Fannie Mae—the other GSE involved in the secondary mortgage market—has undergone changes nearly identical to those found in Figure 9. This has occurred even though the two GSEs possess separate underwriting guidelines. Fannie Mae’s DTI requirement is more lax than Freddie Mac’s: for loans underwritten using Fannie’s proprietary software, the maximum allowable DTI ratio is 50%. However, these data indicate that the relevant statutory limit in the conforming loan market is the more strict requirement of 45%.

In Figure 17, I plot residualized LTV ratios as a function of DTI ratios separately for loans originated before and after the introduction of the DTI limit in 2009. Prior to the policy, the LTV ratio is initially increasing in the DTI ratio before flattening out. After the policy, loans to the right of the DTI limit have smaller LTV ratios compared to loans just at or below the limit. Additionally, I estimate the difference-in-differences specification in Equation (1) using the LTV ratio as the dependent variable. The coefficient on the interaction term is negative and statistically significant at the 1% level. Table 10 contains estimated values for $\beta_1$ and $\beta_2$ and shows that, relative to low-DTI loans, the LTV ratio of high-DTI loans declined by 5.5 percentage points after the DTI limit came into effect.

66To ensure comparability with the sample of loans purchased by Freddie Mac that I use in my empirical analysis, I only include in this figure loans purchased or guaranteed by Fannie Mae with 30-year terms that are collateralized by owner-occupied housing and have non-missing data on DTI ratios, FICO scores, interest rates, and LTV ratios. This leaves me with around 23 million observations.
Table 9. Loan-level summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>25th pctile.</th>
<th>50th pctile.</th>
<th>75th pctile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI ratio (%)</td>
<td>34.64</td>
<td>10.86</td>
<td>27</td>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>Credit score</td>
<td>744.39</td>
<td>50.54</td>
<td>711</td>
<td>755</td>
<td>785</td>
</tr>
<tr>
<td>Interest rate (%)</td>
<td>5.01</td>
<td>1.02</td>
<td>4.13</td>
<td>4.88</td>
<td>5.88</td>
</tr>
<tr>
<td>LTV ratio (%)</td>
<td>72.81</td>
<td>16.49</td>
<td>65</td>
<td>78</td>
<td>80</td>
</tr>
<tr>
<td>Loan amount (000s)</td>
<td>224.16</td>
<td>115.54</td>
<td>135</td>
<td>200</td>
<td>297</td>
</tr>
</tbody>
</table>

Notes: The DTI ratio, credit score, LTV ratio, and loan amount at origination are reported as whole numbers in the dataset. Source: Freddie Mac Single Family Loan-Level Dataset.

Figure 8. Loan characteristics over time

Notes: The vertical dashed black line at 2009 indicates the introduction of the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.
Figure 9. Distribution of DTI ratios of loans purchased by Freddie Mac

Notes: Mortgages are grouped into 1-percentage point bins. The dashed black vertical line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.
Figure 10. Distribution of DTI ratios of loans purchased by Fannie Mae

Notes: Mortgages are grouped into 1-percentage point bins. The dashed black vertical line indicates the Freddie Mac 45% DTI limit. Source: Fannie Mae Single-Family Loan Performance Data.
Figure 11. Loan size as a function of the DTI ratio at origination

Notes: The LTV ratio is residualized with respect to a vector of quarter dummy variables. Averages are computed for each 1-percentage point DTI bin for pre- and post-2009Q1 observations separately. The vertical dashed black line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.

Table 10. Effect of the 45% DTI limit on LTV ratios at origination

<table>
<thead>
<tr>
<th></th>
<th>LTV ratio (%)</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTI &gt; 45%</td>
<td>-0.073</td>
<td>(0.105)</td>
</tr>
<tr>
<td>DTI &gt; 45% × Policy</td>
<td>-5.363***</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

N: 3,061,386  
R²: 0.238

p-level in parentheses.  
Robust standard errors are clustered at the state level.  
* p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The two rows show estimated values for the coefficients β₁ and β₂ from the baseline difference-in-differences specification in Equation (1). Source: Freddie Mac Single Family Loan-Level Dataset.
A.2 Flexible difference-in-differences specification

To ensure that the results from Section 3.2 are not simply driven by the fact that the DTI ratio on a given loan is large but, rather, a change exactly at the 45% limit, I follow DeFusco, Johnson and Mondragon (2020) in estimating a more flexible difference-in-differences specification that allows the effect of the policy to vary with the DTI ratio. The regression equation is

$$y_{it} = \alpha + \sum_{k=40}^{50} \left[ \beta_1^k \mathbb{1}_{DTI_i=k} + \beta_2^k \mathbb{1}_{DTI_i=k} \times Policy_t \right] + \gamma_t + X_i'\delta + \varepsilon_{it},$$  \hspace{1cm} (18)

where $\mathbb{1}_{DTI_i=k}$ is an indicator variable that takes a value of 1 if the DTI ratio (in percent) of loan $i$ at origination is equal to $k \in \{41, 42, \ldots, 50\}$ and all other terms are as previously defined in Equation (1). I make $k = 45$ the omitted category such that $\beta_2^k$ estimates the differential change in $y_{it}$ for loans originated with a DTI ratio equal to $k$ relative to loans with a DTI ratio of 45% after the policy is introduced. I estimate Equation (18) for a sub-sample of loans with DTI ratios between 40% and 50%, and robust standard errors are clustered at the state level.

Figure 12 plots point estimates for $\{\beta_2^k\}_{k=40}^{50}$ and their respective 95% confidence intervals when Equation (18) is estimated with the credit score as the dependent variable. Relative to borrowers with DTI ratios equal to 45%, the credit score of borrowers who receive high-DTI mortgages increases by 14 after the DTI limit is introduced. Figure 13 plots point estimates for coefficients on the interaction terms and their respective 95% confidence intervals using the interest rate as the dependent variable. Relative to borrowers with DTI ratios equal to 45%, borrowers with high-DTI mortgages receive interest rates that are on average 7 basis points higher after the DTI limit is introduced.

Note that the differential changes in default risk estimated using the flexible difference-in-differences specification are larger than estimated using the baseline specification in Section 3.2. This is because the control group in the baseline specification contains all loans with DTI ratios between 40% and 45%, whereas, in this instance, the control group only consists of borrowers exactly on the limit. As already shown in Figure 2, these borrowers have the highest default risk in the sample.
Figure 12. Effect of the 45% DTI limit on credit scores at origination

Notes: Blue squares correspond to estimates of $\beta_k$ for $k \in \{40, 41, \ldots, 50\}$ from the flexible difference-in-differences specification in Equation (18), where $k = 45$ is the omitted category. Blue bars denote the 95% confidence interval around the point estimates. Source: Freddie Mac Single Family Loan-Level Dataset.
Figure 13. Effect of the 45% DTI limit on interest rates at origination

Notes: Blue squares correspond to estimates of $\beta_k$ for $k \in \{40, 41, \ldots, 50\}$ from the flexible difference-in-differences specification in Equation (18), where $k = 45$ is the omitted category. Blue bars denote the 95% confidence interval around the point estimates. Source: Freddie Mac Single Family Loan-Level Dataset.
B Model appendix

B.1 Parameterizing the income process

To parameterize the deterministic age-dependent component of income, I regress a quartic polynomial in age on log annual household income of households whose heads are between ages 25–65 from the 1999–2017 waves of the Panel Study of Income Dynamics (PSID). I use the estimated coefficients to calculate the fitted values for log household income. These fitted values are the sequence \( \{\chi_j\}_{j=1}^{T_R-1} \). I normalize them such that the log income of age-1 households in the model \( \chi_1 \) is 0.

To parameterize the function for pension income \( \Phi(y_{T_R-1}(z)) \), I follow the procedure from Guvenen and Smith (2014). For a given \((\rho_z, \sigma_\varepsilon)\) pair, I simulate the earnings of a panel of 100,000 households during their working years and regress their average labor earnings on earnings in the last period of working life. I use the regression coefficients to predict average lifetime earnings \( \log \hat{y} \) for each possible realization of income in the last period of working life in the model, \( \log y_{T_R-1}(z) = \chi_{T_R-1} + z \). Letting \( \log \bar{y} \) be the economy-wide average annual labor earnings and defining \( \log \tilde{y} = \log \hat{y} / \log \bar{y} \), pension income is given by

\[
\Phi(y_{T_R-1}(z)) = \begin{cases} 
\log \bar{y} [0.9 \log \bar{y}] & \text{if } \log \tilde{y} \leq 0.3 \\
\log \bar{y} [0.27 + 0.32 (\log \tilde{y} - 0.3)] & \text{if } 0.3 < \log \tilde{y} \leq 2 \\
\log \bar{y} [0.81 + 0.15 (\log \tilde{y} - 2)] & \text{if } 2 < \log \tilde{y} \leq 4.1 \\
1.13 \log \bar{y} & \text{if } \log \tilde{y} > 4.1.
\end{cases}
\]

Figure 14 plots the median life-cycle income profile that results from both procedures.

B.2 Estimating the DTI offset

According to the Freddie Mac Single-Family Seller/Servicer Guide, liabilities that must be considered when computing a borrower’s debt payment-to-income ratio include their monthly housing expense, payments on all installment debts (e.g., student loans), payments on revolving accounts (e.g., credit cards), child support, and alimony. Monthly housing expense consists of principal and interest payments on the mortgage, property hazard insurance premiums, real estate taxes, homeowners association (HOA) dues, and other expenditures where applicable.\(^{67}\)

I directly calibrate the DTI offset from micro data. The offset is the sum of (1) non-first mortgage debt service payments and (2) other housing expenses, both scaled relative

\(^{67}\)See https://guide.freddiemac.com/app/guide/content/a_id/1000663.
Notes: This figure plots median deterministic age-specific component of income \( \exp(\chi_j) \) received between ages 1–40 in the model (equivalent to ages 25–64 in the data), followed by the median pension income \( \Phi(y_{T-1}(z)) \) received between ages 41–56 in the model (equivalent to ages 65–80 in the data).

to income. I use the 2016 Survey of Consumer Finances (SCF) to parameterize the first component of the DTI offset. A household’s non-mortgage DTI ratio is the sum of their non-mortgage revolving and non-revolving ratios; DTI ratios on second and third mortgages; and the ratio of alimony payments to income. To control for the effect that age, household balance sheets, and other household observables may have on the non-mortgage DTI ratio, I estimate the regression

\[
\varsigma_i = \alpha + \sum_{j=1}^{4} \beta_j age_i + \gamma \left( \frac{w_i}{y_i} \right) + \delta \left( \frac{a_i}{y_i} \right) + \zeta 1_{h>0} + X_i' \eta + \varepsilon_i, \tag{19}
\]

where \( \varsigma_i \) is the non-mortgage DTI ratio of household \( i \), \( \alpha \) is a constant term, \( \{age_i\}_{j=1}^{4} \) is a quartic polynomial in age, \( w_i/y_i \) is the net worth to income ratio, \( a_i/y_i \) is the liquid wealth to income ratio, \( 1_{h>0} \) is an indicator variable for homeownership, \( X_i \) is a vector of household demographic characteristics, and \( \varepsilon_i \) is an error term. I include these three balance sheet variables in the regression because they have clear model equivalents. I report estimated regression coefficients in Table 11.

To parameterize the second component of the DTI offset, I use the 2017 American Housing
Table 11. Estimated coefficients from the DTI offset regression equation

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-mortgage DTI ratio</td>
<td></td>
</tr>
<tr>
<td>age</td>
<td>−0.024* (0.060)</td>
</tr>
<tr>
<td>age²</td>
<td>0.001** (0.043)</td>
</tr>
<tr>
<td>age³</td>
<td>−0.000** (0.025)</td>
</tr>
<tr>
<td>age⁴</td>
<td>0.000** (0.014)</td>
</tr>
<tr>
<td>w/y</td>
<td>0.000*** (0.008)</td>
</tr>
<tr>
<td>a/y</td>
<td>−0.001*** (0.000)</td>
</tr>
<tr>
<td>1_{h&gt;0}</td>
<td>−0.011*** (0.000)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.410*** (0.005)</td>
</tr>
</tbody>
</table>

N  16,419
R²  0.035

p-level in parentheses.
* p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The rows report estimated values of \( \{\beta_j\}_{j=1}^4 \), \( \gamma \), \( \delta \), \( \zeta \), and \( \alpha \) from the DTI offset regression in Equation (19). Source: 2016 SCF.

Survey (AHS). Housing expenses relative to income are the sum of monthly property taxes (PROTAXAMT), home insurance premiums (INSURAMT), HOA fees (HOAAMT), and lot rent (LOTAMT), all scaled by monthly household income (HINCP). For simplicity, I assume that all agents in the model have the same housing expense ratio and set it equal to the average in the data, which is 8%.

When solving the problem of a household who chooses to own and obtain a new mortgage loan in Equation (10), I use the estimated values of \( \alpha \), \( \{\beta_j\}_{j=1}^4 \), \( \gamma \), \( \delta \), and \( \zeta \) to predict the non-mortgage DTI ratio given the household’s age and state. I add to that the housing expense ratio calculated from the AHS. In Figure 15, I plot borrowers’ DTI offsets generated by this procedure in the stationary distribution of the model by age. The offset is highest for young households and declines as households age. On average, the DTI offset is 20.1%.
Notes: Holding age $j$ constant, the mean DTI offset is calculated by integrating $\varsigma_j (\omega)$ over the stationary distribution of households who obtain a new loan in their current state.

B.3 Calculating calibration targets from the Survey of Consumer Finances

I use the 2016 SCF summary extract file to compute cross-sectional and life-cycle moments of household balance sheets targeted in the calibration. Net worth is the sum of liquid assets and home equity. Following Kaplan and Violante (2014), liquid assets are the sum of assets held in checking accounts, savings accounts, call accounts, directly held mutual accounts, directly held bonds, and directly held stocks. Home equity is the difference between the value of primary residential real estate and debt outstanding on the first mortgage secured by the primary residence.$^{68}$ I define household income as the sum of wage and salary income; income from unemployment insurance and benefits; and Social Security and pension income. I limit my sample to households whose heads are between ages 25–80 and have strictly positive household income.

All means are calculated with SCF sample weights, and I exclude households in the top 1% of the net worth distribution. The SCF over-samples households who are likely to be relatively wealthy in order to increase representation of the upper tail of the wealth distribution and to make possible analyses of less widely held asset classes (e.g., direct holdings of government bonds). These families correspond to the “list sample” because they

$^{68}$Because relatively few households in the SCF report having more than one loan secured by their primary residence or owning a second home, however, the inclusion of these second mortgages (e.g., home equity loans or home equity lines of credit) is quantitatively unimportant for my results.
are selected using specially edited individual tax returns provided by the Internal Revenue Service. In the 2004 SCF, for example, the list sample accounts for only 15% of observations in the bottom 95% of the wealth distribution but 88% of observations in the top 5% percent.\textsuperscript{69}

### B.4 Additional results on borrower selection into high-DTI mortgages

Figure 16 compares the \textit{ex ante} default probability of borrowers as a function of their DTI ratio in the model to their nearest empirical counterpart, the \textit{ex post} probability that a borrower is delinquent on their mortgage one year after payments begin.\textsuperscript{70} For the originations in my sample, I use the monthly performance files of the Freddie Mac Single Family Loan-Level Dataset to create an indicator variable that takes a value of 1 if a borrower is at least 30 days delinquent on their loan one year after their first payment is due. In reality, borrowers are often delinquent on their payments for some time before a formal foreclosure process is initiated. In the model, delinquency and default are identical decisions: a debtor who does not make a mortgage payment necessarily loses their house as well. Because the definitions of default in the model and the data differ, one should not expect the quantitative magnitudes to line up exactly. Nevertheless, the model generates predictions that are qualitatively similar to what is found in the data.\textsuperscript{71}

In Figure 17, I compare LTV ratios at origination in the model and the data for the pre- and post-DTI limit periods. In line with the data, the model predicts that, with the current DTI limit in place, the LTV ratios of loans above the limit are smaller than those of loans just on the limit. Without the DTI limit, LTV ratios remain elevated past the 45% cutoff. There is much greater dispersion in LTV ratios at origination in the model than in the data. This likely reflects the fact that mortgages are the only form of debt available to households, whereas, in reality, households can use unsecured borrowing to insure against smaller negative shocks to cash on hand. As a result, the model over-predicts the share of households who obtain mortgages with small balances at origination.\textsuperscript{72} This could be addressed by relaxing the no-borrowing constraint on the liquid asset and calibrating the limit on liquid debt to match the observed share of households with positive unsecured debt.

\textsuperscript{70}I do this because the model does not contain a formal notion of a credit score—which is a backward-looking measure of a borrower’s expected ability to repay—\textit{per se}.
\textsuperscript{71}This result is robust to using more or less more or less strict definitions of default—e.g., delinquent at any point in the life of the loan, more than 360 days delinquent, property has been repossessed by the lender—to construct the \textit{ex post} probability of default in the data.
\textsuperscript{72}The mean LTV ratio in the model is 63% versus 76% in the data. However, the distribution of LTV ratios at origination in the model exhibits a mass point at the statutory maximum of 85%, similar to the spikes at institutional LTV limits documented by Greenwald (2018).
**Figure 16.** Default probabilities as a function of DTI ratio at origination in the model vs. data

![Graph showing default probabilities](image1)

**Notes:** “With DTI limit” refers to the baseline calibration of the model. “Without DTI limit” refers to an alternative calibration in which the default risk threshold for a high-DTI loan is fully relaxed. Default probability in the model is the *ex ante* probability that an age-$j$ household in state $\omega$ optimally defaults at age $j+1$. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.

**Figure 17.** LTV ratio as a function of the DTI ratio at origination in the model vs. data

![Graph showing LTV ratio](image2)

**Notes:** “With DTI limit” refers to the baseline calibration of the model. “Without DTI limit” refers to an alternative calibration in which the default risk threshold for a high-DTI loan is fully relaxed. “LTV ratio” refers to LTV ratio at origination. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.
B.5 Additional results on the heterogeneity of default risk at the DTI limit

In Figure 18, I show that borrowers who have DTI ratios between 43% and 45% in the stationary distribution of the model are non-uniform in their likelihood of default. Conditional on choosing a low-DTI loan, around 17% of such borrowers have an expected probability of default that, all else equal, would qualify them for a high-DTI loan. This is similar in magnitude to the overlap in credit scores documented in the data (see Figure 3). Note that, by design, the model restricts the variance of default risk among high-DTI borrowers compared to what is found in the data.

In Figure 19, I map the observed default risk of borrowers found at the DTI limit to their underlying states. To do so, I separate them into two groups based on whether their likelihood of default is less or greater than the qualifying default risk threshold for a high-DTI loan ("safe bunched" and "risky bunched," respectively). I also include high-DTI borrowers for comparison. This figure shows that "safe bunched" borrowers look more like their counterparts above the DTI limit. "Risky bunched" borrowers share more characteristics with households who are constrained along the extensive margin by the DTI limit. These differences in household characteristics are also reflected in the loan size and interest rate received by the different subgroups.

B.6 Calibrating the cost of default on a high-DTI loan under Dodd-Frank

The Dodd-Frank Act of 2010 delegated the implementation of the ability-to-repay rule to the newly created Consumer Financial Production Bureau (CFPB). In January 2013, the Consumer Financial Protection Bureau (2013) announced the final rule and officially entered it into the Federal Register. I follow a cost-benefit analysis from the final rule to calibrate the value of $\gamma_H$, the cost of default on a high-DTI loan to lenders.\(^7^3\)

If a borrower who defaults on a high-DTI mortgage brings a successful legal claim under the ability-to-repay rule, the lender who originated that mortgage is liable for up to three years of fees and finance charges; the borrower’s legal expenses; and statutory damages under the Truth in Lending Act (TILA).\(^7^4\) Borrowers who default on a low-DTI mortgage do not have standing under this rule to sue the lender; thus, I leave the calibrated value of $\gamma_L$ unchanged. To compute the fees and finance charges owed by the lender, I calculate

\(^7^3\)The final rule can be found at https://www.federalregister.gov/documents/2013/01/30/2013-00736/ability-to-repay-and-qualified-mortgage-standards-under-the-truth-in-lending-act-regulation-z.

\(^7^4\)The Truth in Lending Act of 1968 requires that lenders disclose credit terms to consumers in a “meaningful way.” Failure to do so can result in the lender being liable for statutory damages. The ability-to-repay rule is implemented through TILA, and the Dodd-Frank Act authorized the transfer of TILA’s rule-making authority from the Federal Reserve to the CFPB. See https://www.consumerfinance.gov/policy-compliance/rulemaking/regulations/1026/.
Figure 18. Distribution of default risk in the model

Notes: The dashed vertical black line indicates the calibrated value of the qualifying default risk threshold for a high-DTI loan, $\Psi = 0.025\%$.

Figure 19. High-DTI and bunched borrowers in the model

Notes: “Safe (risky) bunched” refers to borrowers who optimally choose a low-DTI loan and have an endogenous default probability less (greater than) than $\Psi = 0.025\%$. “High DTI” refers to borrowers who optimally choose a high-DTI loan. Home equity and net worth are scaled by income. Loan size is in units of the numeraire.
I then add to that the mortgage origination cost $\kappa_m$. The CFPB estimates that combined legal expenses of a lender and borrower are $34,500 and additionally assumes that a borrower is rewarded $4,000 in statutory damages under TILA. Together, the legal costs amount to 74% of mean U.S. household income, equivalent to 1.03 units of the numeraire. Finally, I add to this the housing transaction cost $\kappa_h$ to account for costs generated by the foreclosure itself. In total, the resource loss suffered by a lender due to default on a high-DTI loan, $\gamma_H$, is 2.15. This amounts to 46% of the average loan size at origination and is about 4.5 times the cost of default on a low-DTI loan to lenders.

I show in Table 12 that results from my policy counterfactual are robust to different values for $\gamma_H$. In reality, not every borrower who is unable to repay their mortgage will bring a case against the responsible lender. That decision depends on, among other factors, whether the borrower lives in a judicial or non-judicial foreclosure state and their willingness and/or ability to obtain legal representation. Substantial evidence nonetheless suggests that mortgage lenders are indeed worried about these regulations. Fuster, Lo and Willen (2017) estimate that, between 2008 and 2014, the price of intermediation in the mortgage market increased by around 30 basis points per year and that this trend appears to be driven by increased net costs of mortgage servicing and heightened aversion to liability risk among lenders. Kim et al. (2018) document legal actions the GSEs and the U.S. federal government

<table>
<thead>
<tr>
<th>$\gamma_H$</th>
<th>CEV (%)</th>
<th>Share with CEV $\geq 0$</th>
<th>Share with CEV $&lt; 0$</th>
<th>Own. rate</th>
<th>Def. rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.15</td>
<td>1.31</td>
<td>0.87</td>
<td>0.13</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>0.48</td>
<td>1.40</td>
<td>0.89</td>
<td>0.11</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>1.08</td>
<td>1.39</td>
<td>0.88</td>
<td>0.12</td>
<td>0.71</td>
<td>0.78</td>
</tr>
<tr>
<td>1.68</td>
<td>1.35</td>
<td>0.85</td>
<td>0.15</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>2.28</td>
<td>1.30</td>
<td>0.85</td>
<td>0.15</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>2.88</td>
<td>1.23</td>
<td>0.84</td>
<td>0.16</td>
<td>0.71</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Notes: “CEV” is consumption-equivalent welfare change. “Own. rate” and “def. rate” are homeownership and default rates in the model under the Dodd-Frank reform, respectively.
took after the Great Recession in response to improper loan originations. In light of this, the fact that Dodd-Frank creates the potential for increased legal claims against mortgage lenders—even if there is uncertainty about how many will claims will ultimately be brought—should be taken seriously.

B.7 Defining consumption-equivalent welfare change

Following Gete and Zecchetto (2018), I define the composite consumption of an age-$j$ household as

$$C_j \equiv \left( \alpha_j c_{j-\vartheta}^1 + (1 - \alpha_j) s_{j-\vartheta}^1 \right)^{\frac{1}{1-\sigma}}.$$  

Consumption-equivalent welfare change $\Delta C_j (\omega)$ is the percent by which the composite consumption of an age-$j$ household in state $\omega$ would need to change to make them exactly indifferent between the stationary economies under the current and proposed policies. The assumption of CES preferences yields the closed-form solution

$$\Delta C_j (\omega) = \left[ \left( \frac{\tilde{V}_j (\omega)}{V_j (\omega)} \right)^{\frac{1}{1-\sigma}} - 1 \right] \times 100,$$

where $V_j (\omega)$ and $\tilde{V}_j (\omega)$ are the value functions of an age-$j$ household in state $\omega$ under the current and new policies, respectively. Aggregate consumption-equivalent welfare is therefore

$$\Delta C = \int \Delta C_j (\omega) \Lambda_j (d\omega),$$

where $\Lambda_j (\omega)$ is the stationary distribution of households over states under the initial policy.

B.8 Varying the degree of lender discretion under Dodd-Frank

Table 13 presents the effects of the Dodd-Frank reform under different values for $\Psi$, the default risk threshold for a high-DTI loans. The first row corresponds to an always binding default risk threshold ($\Psi = 0\%$) and is isomorphic to a model in which all mortgage originations are subject to the DTI limit. The last row corresponds to an always relaxed default risk threshold ($\Psi = 100\%$) and is isomorphic to a model in which no mortgage originations are subject to the DTI limit. The second row corresponds to a case in which lender discretion remains unchanged as a result of the Dodd-Frank reform ($\Psi = 0.025\%$) but the other features of the policy—the tighter DTI requirement and the higher cost of originating a high-DTI loan—are implemented in the model as is. The fifth row contains the parameterization ($\Psi = 3.55\%$) used in the main text of the paper.
Table 13. Effects of Dodd-Frank reform under different values of the default risk threshold $\Psi$

<table>
<thead>
<tr>
<th>$\Psi$ (%)</th>
<th>CEV (%)</th>
<th>Share with CEV $\geq$ 0</th>
<th>Share with CEV $&lt; 0$</th>
<th>Own. rate</th>
<th>Def. rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>-1.20</td>
<td>0.08</td>
<td>0.92</td>
<td>0.65</td>
<td>0.58</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.67</td>
<td>0.13</td>
<td>0.87</td>
<td>0.66</td>
<td>0.58</td>
</tr>
<tr>
<td>1.00</td>
<td>0.24</td>
<td>0.70</td>
<td>0.30</td>
<td>0.68</td>
<td>0.61</td>
</tr>
<tr>
<td>2.00</td>
<td>0.35</td>
<td>0.73</td>
<td>0.27</td>
<td>0.68</td>
<td>0.62</td>
</tr>
<tr>
<td>3.55</td>
<td>1.31</td>
<td>0.85</td>
<td>0.15</td>
<td>0.71</td>
<td>0.79</td>
</tr>
<tr>
<td>5.00</td>
<td>3.93</td>
<td>0.97</td>
<td>0.03</td>
<td>0.76</td>
<td>1.30</td>
</tr>
<tr>
<td>100</td>
<td>3.94</td>
<td>0.98</td>
<td>0.02</td>
<td>0.76</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Notes: “CEV” is consumption-equivalent welfare change. “Own. rate” and “def. rate” are homeownership and default rates in the model under the Dodd-Frank reform, respectively.

C Numerical solution of the model

C.1 Simplifying the state space

Following Berger and Vavra (2015) and Kaplan and Violante (2014), I redefine the state space of the model by noting that, conditional on the household choosing to adjust their house size, it is sufficient to track their home equity. In other words, the household only cares about the net cash on hand that results from selling their existing house and repaying their outstanding mortgage debt, not its composition. I define the home equity $e$ of a household with house size $h$, house price $p$, depreciation shock $\delta$, mortgage debt $m$, and interest rate $r_m$ as

$$e(h, \delta, p, m, r_m) \equiv (1 - \delta) ph - (1 + r_m) m. \tag{20}$$

The cash-on-hand formulation significantly reduces the dimensionality of the state vector of a household who chooses to rent or chooses to obtain a new mortgage. Because the optimization problem associated with the latter is the most computationally intensive part of solving the household’s problem, this dramatically reduces overall computation time. Let $\omega_A \equiv (a, h, e, p, z)$ denote the current state of an age-$j$ household solving either of those optimization problems.\(^78\)

A household who continues with an existing mortgage does not adjust their house size, and it remains necessary to track their outstanding debt, mortgage interest rate, and contract type separately. As a result, I define their optimization problem in terms of the original vector of state variables in Section 4.2. For notational consistency, however, I relabel the state vector

\(^78\)Tracking $h$ as a separate state variable is necessary for determining if a household pays the housing adjustment cost. Tracking the depreciation shock $\delta$ is not, both because it is subsumed in the definition of home equity in Equation (20) and i.i.d. by assumption.
of an age-\(j\) household who continues with an existing loan as \(\omega_N \equiv (a, h, \delta, p, m, q, r_m, z)\).

Finally, note that, conditional on choosing to default, home equity becomes irrelevant to the household’s problem because default, by definition, requires them to surrender their house and sets their outstanding debt to zero. However, it is still necessary to track the type of mortgage contract because the household’s decision to default affects the expected discounted cash flow associated with the loan. Thus, I define the state vector of an age-\(j\) household who chooses to default as \(\omega_D \equiv (a, p, q, z)\).

### C.2 Redefining optimization problems

I rewrite the household’s optimization problem for the redefined state space. The expected discounted lifetime utility of an age-\(j\) household who rents in state \(\omega_A\) is

\[
W_j^R(\omega_A) = \max_{x_j(\omega_A)} \{ V_j^R(\omega_A), V_j^M(\omega_A) \}.
\]  

(21)

The expected discounted lifetime utility of an age-\(j\) household who owns in state \(\omega_N\) is

\[
W_j^O(\omega_N) = \max_{x_j(\omega_N)} \{ V_j^R(\omega_A), V_j^M(\omega_A), V_j^P(\omega_N), V_j^D(\omega_D) \},
\]  

(22)

where \(\omega_A = (a, h, e(h, \delta, p, m, r_m), p, z)\).

Conditional on choosing to rent, a household solves

\[
V_j^R(\omega_A) = \max_{c, s, a'} u_j(c, s) + \beta \mathbb{E}_{\omega'_{j+1}} W_{j+1}^R(\omega'_{A})
\]

\[\text{st.}\]
\[c + Rps + a' \geq y_j(z) + (1 + r)a + e - \mathbbm{1}_{h \neq 0} \kappa_h\]
\[a' \geq 0\]
\[\omega'_{A} = (a', 0, 0, p', z').\]

(23)

Note that, if a household begins the next period as a renter, taking expectations over the depreciation shock is unnecessary because, by definition, they will have a house size of 0.
Conditional on obtaining a new mortgage, a household solves

\[
V_j^M (\omega_A) = \max_{c,a',h',m',q' \in \{L,H\}} u_j (c,h') + \beta \mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^O (\omega'_N) \\
\text{s.t.} \\
c + a' + \mathbbm{1}_{h' \neq h} (ph' - \kappa_h) \leq y_j (z) + (1 + r) a + e - (1 - \mathbbm{1}_{h' \neq h}) ph + m' - \mathbbm{1}_{m' > 0} \kappa_m \\
m' \leq \theta ph' \\
\pi_{min,j} (m',r_{m,j}' (\omega_A)) \leq \lambda_j (q') y_j (z) \\
\lambda_j (q') = \begin{cases} 
\lambda - \varsigma_j (\omega_A) & \text{if } q' = L \\
\infty & \text{if } q' = H \text{ and } \psi_j (\omega_A) < \Psi
\end{cases} \\
a' \geq 0 \\
\omega'_N = (a',h',\delta',p',m',q',r_{m,j}' (\omega_A),z').
\] (24)

This particular formulation of the flow budget constraint allows me to compute the housing maintenance costs and loan repayment required by a mortgage refinance using this lower-dimensional state vector, despite the household not adjusting their housing stock. A mortgage refinance occurs when \( \mathbbm{1}_{h' \neq h} = 0 \) and \( \mathbbm{1}_{m' > 0} = 1 \). Using the definition of home equity in Equation (20), the household’s cash on hand is

\[
y_j (z) + (1 + r) a - \delta ph - (1 + r_m) m + m' - \mathbbm{1}_{m' > 0} \kappa_m.
\]

Therefore, knowledge of \( h, e, \) and \( p \) is sufficient for backing out the housing maintenance costs \( \delta ph \) and loan repayment \( (1 + r_m) m \).

Conditional on making a payment on an existing loan, a household solves

\[
V_j^P (\omega_N) = \max_{c,a,m} u_j (c,h) + \beta \mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^R (\omega'_N) \\
\text{s.t.} \\
c + \delta ph + a' \leq y_j (z) + (1 + r) a - (1 + r_m) m + m' \\
m' \leq (1 + r_m) m - \pi_{min,j} (m,r_m) \\
a' \geq 0 \\
\omega'_N = (a',h,\delta',p',m',q,r_m,z').
\] (25)
Conditional on defaulting, a household solves

\[ V^D_j(\omega_D) = \max_{c,s,a'} \left[ \varphi \mathbb{E}_{\omega',z'|p,z} \left[ \varphi W^R_j(\omega_A') + (1 - \varphi) V^R_{j+1}(\omega_A') \right] \right] \]

s.t.

\[ c + Rps + a' \leq y_j(z) + (1 + r) a \]
\[ a' \geq 0 \]
\[ \omega_A' = (a', 0, 0, p', z'). \]

The financial intermediary’s problem presented in Section 4.3 remains unchanged. For notational consistency, I rewrite the present value of an existing mortgage held by an age-\( j \) household in state \( \omega_N \) as

\[ \Pi_j(\omega_N) = \begin{cases} (1 + r_m) m & \text{if repay} \\ (1 - \delta) ph - \gamma(q) & \text{if default} \\ (1 + r_m) m - m'_j(\omega_N) + \frac{1}{1 + r + \theta} \mathbb{E}_{\omega',z'|p,z} \Pi_{j+1}(\omega_N') & \text{otherwise}, \end{cases} \]

where \( \omega_N' = (a'_j(\omega_N), h'_j(\omega_N), \delta', p', m'_j(\omega_N), q'_j(\omega_N), r'_{m,j}(\omega_N), z') \). I rewrite the zero-profit condition on a mortgage originated to an age-\( j \) household in state \( \omega_A \) as

\[ m'_j(\omega_A) = \frac{1}{1 + r + \theta} \mathbb{E}_{\omega',z'|p,z} \Pi_{j+1}(\omega_N'), \]

where \( \omega_N' = (a'_j(\omega_A), h'_j(\omega_A), \delta', p', m'_j(\omega_A), q'_j(\omega_A), r'_{m,j}(\omega_A), z'). \)

C.3 Discretization

I use the Rouwenhorst method described by Kopecky and Suen (2010) to discretize the first-order Markov processes for shocks to income \( z \) and house prices \( p \). This method yields grids for each shock and a unique matrix of transition probabilities. The grid for house size \( h \) is \( \{0, \bar{h}, \ldots, \bar{h}\} \), where, by definition, \( h = 0 \) for existing renters and \( h \in \{\bar{h}, \ldots, \bar{h}\} \) for existing owners. The points on the housing grid are set according to the procedure described in Section 5.2. The bounds on the grid for mortgage debt \( m \) are \([0, \theta \bar{p} \bar{h}]\), where \( \bar{p} \) is the maximum possible house price. The bounds on the grid for the mortgage interest rate \( r_m \) are \([r + \phi, \bar{r}_m]\). I confirm \textit{ex post} that, in stationary equilibrium, the upper bound \( \bar{r}_m \) does not bind. The grid for the depreciation shock \( \delta \) is \( \{\bar{\delta}, \delta\} \). The grid for the mortgage contract \( q \) is \( \{L, H\} \). The grid for liquid assets \( a \) features more points clustered near the borrowing constraint. The bounds on the liquid asset grid are \([0, \bar{a}]\). I set \( \bar{a} \) to \((1 - \delta) \bar{p} \bar{h} + a_{\text{mult}} \bar{y}_j(z), \)
where \( a_{\text{malt}} > 0 \) is a scalar and \( \bar{y}_j(z) \) is the largest possible income realization in the model. I confirm ex post that the upper bound on liquid assets is not binding in the stationary distribution of the model. The bounds on the grid for home equity \( e \) are \([\underline{e}, \bar{e}]\), where

\[
\underline{e} = (1 - \delta) \bar{p} \bar{h} - (1 + \bar{r}_m) \bar{m} \quad \text{and} \quad \bar{e} = (1 - \delta) \bar{p} \bar{h} - (1 + r_m) \bar{m}.
\]

The home equity grid contains a point at 0.

Choices for the three endogenous state variables \( a, m, \) and \( r_m \)—as well as values of \( e \) where applicable—are permitted to lie off the grid. I use linear interpolation to evaluate the value and policy functions at off-grid points when solving the model. I constrain the choice of \( h' \) to the house size grid in order to capture the indivisibility of housing as an asset. To construct the transition function for households over states \( Q_j(\omega_N, \Omega_N) \) and compute the stationary distribution \( \Lambda^*_j(\Omega_N) \), I interpolate the household’s policy and value functions over finer grids for the three continuous endogenous state variables.

To evaluate expectations inside of the Bellman equations, I pre-compute integrals using the technique described by Judd et al. (2017). After I finish solving an age-\( j \) household’s problem for all \( \omega_A, \omega_D, \) and \( \omega_N \), I define and compute the following for \( j < T \):

\[
\begin{align*}
EW^R_j(\omega_A) &\equiv \mathbb{E}_{\theta', \omega' | p, z} \max \{ V^R_j(a, h, e, p', z'), V^M_j(a, h, e, p', z') \} \\
EW^O_j(\omega_N) &\equiv \mathbb{E}_{\delta', \theta', \omega' | p, z} \max \{ V^R_j(a, h, \bar{e}', p', z'), V^M_j(a, h, \bar{e}', p', z') , \omega' \}
\end{align*}
\]

where \( \bar{e}' \equiv (1 - \delta') \bar{p}' h - (1 + r_m) m \). \( EW^R_j(\omega_A) \) and \( EW^O_j(\omega_N) \) represent the household’s continuation values conditional on holding the endogenous state variables fixed. Therefore, I compute the value functions of an age-\( j \) household as

\[
\begin{align*}
V^R_j(\omega_A) &= u(c^R_j(\omega_A), s^R_j(\omega_A)) + \beta EW^R_{j+1}(a^R_{j+1}(\omega_A), 0, 0, p, z) \\
V^M_j(\omega_A) &= u(c^M_j(\omega_A), h^M_j(\omega_A)) \\
&\quad + \beta EW^O_{j+1}(a^M_{j+1}(\omega_A), h^M_j(\omega_A), \delta, p, m^M_j(\omega_A), q^M_j(\omega_A), r^M_{m,j}(\omega_A), z), \\
V^P_j(\omega_N) &= u(c^P_j(\omega_N), h) + \beta EW^O_{j+1}(a^P_{j+1}(\omega_N), h, \delta, p, m^P_j(\omega_N), q, r_m, z), \\
V^D_j(\omega_D) &= u(c^D_j(\omega_D), s^D_j(\omega_D)) + \beta [\varphi EW^R_{j+1}(a^D_{j+1}(\omega_D), 0, 0, p, z) \\
&\quad + (1 - \varphi) EW^R_{j+1}(a^D_{j+1}(\omega_D), 0, 0, p, z)], \\
\end{align*}
\]

where continuation values are obtained by interpolating \( EW^R_{j+1}(\omega_A) \) and \( EW^O_{j+1}(\omega_N) \) at values for the endogenous state variables in the next period. I also pre-compute integrals for the financial intermediary’s problem, defining

\[
\begin{align*}
E\Pi_j(\omega_N) &\equiv \mathbb{E}_{\delta', \theta', \omega' | p, z} \Pi_j(a, h, \delta', p', m, q, r_m, z'), \\
\end{align*}
\]
such that the zero-profit condition is computed as
\[
m^M_j(\omega_A) = \frac{1}{1 + r + \phi} E \Pi_{j+1} \left( a^M_j(\omega_A), h^M_j(\omega_A), \delta, p, m^M_j(\omega_A), q^M_j(\omega_A), r^M_{m,j}(\omega_A), z \right). \]

### C.4 Solution algorithm

1. Solve the problem of a household in the last period of life to obtain \( V^R_T(\omega_A), V^M_T(\omega_A), V^P_T(\omega_N), \) and \( V^D_T(\omega_D) \), along with all associated policy functions. By assumption, \( m'_T(\omega_N) = 0 \) and \( m'_T(\omega_A) = 0 \). Next, compute the present value of cash flows associated with a mortgage held by an age-\( T \) household in state \( \omega_N \). If the household repays their debt, then
\[
\Pi_T(\omega_N) = (1 + r_m) m. \]
If the household defaults on their debt, then
\[
\Pi_T(\omega_N) = (1 - \delta) p_h - \gamma(q). \]

2. Use backward induction to solve for value functions in Equations (23), (24), (25), and (26) for ages \( j < T \).

(a) **Solving the problem of a household who chooses to rent, \( V^R_j(\omega_A) \):**

(i) This option is available to all households.

(ii) The assumption of CES preferences over nondurable consumption and housing services implies
\[
s = \left( \frac{1 - \alpha}{\alpha R} \right)^{\frac{1}{\sigma}} c. \tag{29} \]

Use this expression to substitute out \( s \) in the household’s problem, and use the budget constraint to substitute out \( c \) from the flow utility function.

(iii) Solve for \( V^R_j(\omega_A) \) and \( a^R_j(\omega_A) \) using Brent’s method. Let \( \zeta \) be the lowest possible value of nondurable consumption.\(^79\) The requirement that \( c \geq \zeta \) characterizes the set of feasible solutions,
\[
a' \leq a' \leq y_j(z) + (1 + r) a + c - 1_{h \neq 0} k_h - \zeta \left[ 1 + \left( \frac{1 - \alpha}{\alpha R^{1-\sigma}} \right)^{\frac{1}{\sigma}} \right]^{-1}, \]
where the no-borrowing constraint on the liquid asset implies \( a' = 0 \).

\(^79\)In the computation, \( \zeta \) is set to 0.001.
(iv) Find \( c_j^R(\omega_A) \) from the flow budget constraint and use Equation (29) to find \( s_j^R(\omega_A) \).

(v) By definition, \( h_j^R(\omega_A) = 0 \) and \( m_j^R(\omega_A) = 0 \). Because the household is not a debtor, \( q_j^R(\omega_A) \) and \( r_{m,j}^R(\omega_A) \) can be set to any arbitrary value.

(b) Solving the problem of a household who owns and obtains a new loan, \( V_j^M(\omega_A) \):

(i) This option is available to all households. When solving this problem for age-

T households, \( m_j^M(\omega_A) \) is constrained to be 0, there is no mortgage contract
choice to make, and the financial intermediary’s problem is skipped.

(ii) Hold \( q' \) fixed. For a given \( r_{m,j}^2(\omega_A) \), loop through all feasible \( h' \). For each feasible \( h' \), solve for \( a_j^M(\omega_A) \), \( m_j^M(\omega_A) \), and \( V_j^M(\omega_A) \) using Nelder-Mead. Maximum feasible borrowing is determined by the LTV and DTI limits. If \( q' = L \), then

\[
\bar{m}' = \min \{ m'_{ltv}, m'_{dti} \},
\]

where

\[
m'_{ltv} \equiv \theta ph',
\]

and

\[
m'_{dti} \equiv \lambda_j (q') y_j(z) \left[ r'_{m,j}(\omega_A) \left( \frac{(1 + r'_{m,j}(\omega_A))^{T-j}}{(1 + r'_{m,j}(\omega_A))^{T-j} - 1} \right) \right]^{-1},
\]

where \( \lambda_j (q') \) is defined in Equation (4). If \( q' = H \), then

\[
\bar{m}' = m'_{ltv}.
\]

The set of feasible solutions is therefore characterized by

\[
a' \leq a' \leq y_j(z) + (1 + r) a + e - (1 - 1_{h' \neq h}) ph + \bar{m}' - \kappa_m
- 1_{h' \neq h} (ph' + \kappa_h) - c
\]

and

\[
\max \{ c + 1_{h' \neq h} (ph' + \kappa_h) + a' + \kappa_m + (1 - 1_{h' \neq h}) ph - y_j(z)
- (1 + r) a - e, 0 \} \leq m' \leq \bar{m}'.
\]

Use the budget constraint to find \( c_j^M(\omega_A) \). Select the value of \( h' \) (and policy functions implied that by that choice) that yields the highest value for the

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Footnote: In this context, feasibility means that, holding the household’s current state and contract type \( q' \) fixed, \( h' \) is in their budget set and \( c \geq c \) assuming \( m_j^M(\omega_A) = \bar{m}' \), \( a_j^M(\omega_A) = a' \), and \( r_{m,j}^M(\omega_A) = \Sigma m' \).
household.

(iii) Given a solution to the household’s problem found in the previous step, compute the financial intermediary’s profit using Equation (28).

(iv) A bisection algorithm is used to find the break-even interest rate on a newly originated mortgage \( r'_{m,j}(\omega_A) \). This algorithm exploits the fact that the financial intermediary’s profit is increasing in \( r'_{m,j}(\omega_A) \), all else equal, and searches over the interval \([\underline{r}_m, \bar{r}_m]\).

(A) The interest rate received by a household of age \( T - 1 \) who obtains a new loan \( r'_{m,T-1}(\omega_A) \) is \( r + \phi \). This follows from the fact that, if an age-\( T \) household repays their outstanding mortgage debt, then the zero profit condition is

\[
\frac{1}{1 + r + \phi} \left( 1 + r'_{m,T-1}(\omega_A) \right) m'_{T-1} = m'_{T-1},
\]

implying \( r'_{m,T-1}(\omega_A) = r + \phi \). Note that, if an age-\( T \) household defaults, then, in equilibrium, an intermediary will not sell that household a mortgage contract in the previous period.

(B) If the financial intermediary’s profit is negative when \( r'_{m,j}(\omega_A) = \bar{r}_m \), then the option to obtain a mortgage is not available to the household in equilibrium and the expected probability of default \( \psi_j(\omega) \) is set to 1. If the intermediary’s profit is positive when \( r'_{m,j}(\omega_A) = \underline{r}_m \) and negative when \( r'_{m,j}(\omega_A) = \bar{r}_m \), then an interior solution exists and bisection is used to solve for the equilibrium interest rate that earns the lender zero profits on the loan in expectation.

(v) Repeat (ii)–(iv) for each \( q' \). For \( q' = H \), compute the household’s expected probability of default using Equation (11). The household chooses the mortgage contract that yields highest value with the stipulation that a high-DTI loan is only chosen if it is optimal and the expected probability of default on that loan is less than \( \Psi \).

(c) Solving the problem of an owner who continues with an existing loan, \( V^P_j(\omega_N) \):

(i) This option is only available to existing homeowners \((h > 0)\). Note that this problem is also solved by homeowners who do not have any debt. In this case, \( m^P_j(\omega_N) = 0 \) and the owner only needs to solve for \( a^P_j(\omega_N) \).
(ii) Use the budget constraint to substitute $c$ out of the flow utility function.

(iii) Solve for $a_j^P (\omega_N)$, $m_j^P (\omega_N)$, and $V_j^P (\omega_N)$ using Nelder-Mead. From the law of motion for mortgage debt, we have

$$\bar{m}' = (1 + r_m) m - \pi_{\text{min},j} (m, r_m),$$

where $\pi_{\text{min},j} (m, r_m)$ is the minimum mortgage payment defined in Equation (5). The set of feasible solutions is characterized by

$$a' \leq a' \leq y_j (z) + (1 + r) a - (1 + r_m) m + \bar{m}' - \delta ph - c$$

and

$$\max \{c + a' + \delta ph - y_j (z) - (1 + r) a, 0\} \leq m' \leq \bar{m}' .$$

(iv) Use the budget constraint to find $c_j^P (\omega_N)$. By definition, $h_j^P (\omega_N) = h$, $q_j^P (\omega_N) = q$, and $r_{m,j}^P (\omega_N) = r_m$.

(d) Solving the problem of a borrower who defaults, $V_j^D (\omega_D)$:

(i) This option is only available to existing borrowers ($h > 0$ and $m > 0$).

(ii) Because a household who defaults must rent in the current period, use Equation (29) to substitute out $s$ from the flow utility function and the budget constraint.

(iii) Solve for $V_j^D (\omega_D)$ and $a_j^R (\omega_D)$ using Brent’s method. The set of feasible solutions is characterized by

$$a' \leq a' \leq y_j (z) + (1 + r) a - (1 + r_m) m + \bar{m}' - \delta ph - c$$

and

$$\max \{c + a' + \delta ph - y_j (z) - (1 + r) a, 0\} \leq m' \leq \bar{m}' .$$

(iv) Back $c_j^D (\omega_D)$ out from the flow budget constraint and use Equation (29) to find $s_j^D (\omega_D)$.

(v) By definition, $h_j^D (\omega_D) = 0$ and $m_j^D (\omega_D) = 0$. Because the household does not have any mortgage debt, $q_j^D (\omega_D)$ and $r_{m,j}^D (\omega_D)$ can be set to any arbitrary value.

(e) Determine $W_j^R (\omega_A)$ and $W_j^O (\omega_N)$ using Equations (21) and (22).

(f) Compute $\Pi_j (\omega_N)$ using Equation (27).

(g) Compute $EW_j^R (\omega_A)$, $EW_j^O (\omega_N)$, and $E\Pi_j (\omega_N)$.

3. After solving for the stationary recursive equilibrium defined in Section 4.4, interpolate
value functions \( \{V^R_j(\omega_N), V^M_j(\omega_N), V^P_j(\omega_N), V^D_j(\omega_N)\} \) and policy functions for endogenous state variables \( \{a'_x^j(\omega_N), h'_x^j(\omega_N), m'_x^j(\omega_N), q'_x^j(\omega_N), r'_{m,j}(\omega_N)\} \) for all \( x \in \{R, M, P, D\} \) over finer grids for liquid assets \( a \), mortgage debt \( m \), and the interest rate \( r_m \). Determine housing tenure and loan adjustment choices using the interpolated valued functions. Compute policy functions for the control variables accordingly.

4. Given the finer value functions and policy functions for endogenous state variables in the previous step, as well as the probability distributions of \( \delta, p, \) and \( z \), construct the \( (ns_{\text{fine}}/T) \times (ns_{\text{fine}}/T) \) transition matrix for the distribution of households over states \( Q_j(\omega_N, \Omega_N) \) according to to Equation (17), where \( (ns_{\text{fine}}/T) \) denotes the number of fine grid points for a given \( j \).

5. For a given \( \mu_1(\omega_N) \) and \( Q_j(\omega_N, \Omega_N) \), use the law of motion in Equation (16) to compute \( \Lambda^*_j+1(\Omega_N) \) for all \( j \in \{1, 2, \ldots, T-1\} \).

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81 In order to preserve the non-convexities introduced by the discrete choices in this model, I interpolate \( V^M_j(\omega_N) \) and \( \psi_j(\omega_N) \) for each possible \( q' \), then determine \( q'^M_j(\omega_N) \) based on that. Note that, in this step, I define finer policy and value functions over the originally defined state vector \( \omega_N \).