Default Risk Heterogeneity and Borrower Selection in the Mortgage Market*

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Abstract

This paper studies limits on debt payment-to-income (DTI) ratios on mortgages that are hard for some but not all households. I document that, under such a policy, a higher DTI ratio need not imply greater default risk. Consequently, households vary substantially in their ability to repay conditional on their DTI ratios. To rationalize observed patterns of loan choice, I propose an incomplete-markets life-cycle model with endogenous mortgage default in which competitive lenders relax a DTI constraint for households who are sufficiently creditworthy in equilibrium. I use the calibrated model to evaluate the Dodd-Frank ability-to-repay rule, which increases the cost of originating certain loans. A version of the rule that uses a loan's price instead of its DTI ratio as a proxy for default risk increases welfare more because fewer borrowers hold high-price than high-DTI loans in equilibrium, reducing the incidence of the rule's costs.

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1 Introduction

After the Great Recession, policymakers introduced new reforms aimed at reducing mortgage default. In the United States, these reforms included a limit on debt payment-to-income (DTI) ratios in the underwriting standards for conforming loans that has dramatically shaped the allocation of mortgage credit.[1] Lenders in practice have some discretion to relax this DTI limit; however, structural macroeconomic models with endogenous mortgage default typically assume that such ad hoc borrowing constraints are hard. The main goal of this paper is to study the efficacy and welfare effects of "flexible" DTI limits in the presence of realistic household heterogeneity.

I document that, under existing underwriting standards in which a DTI constraint is hard for some households but soft for others, a higher DTI ratio does not necessarily imply higher default risk. To explain this, I incorporate a tractable underwriting technology in an otherwise standard incomplete-markets life-cycle model with a competitive mortgage market whereby lenders relax a DTI constraint for households with sufficiently low endogenous probability of default. The calibrated model rationalizes empirical features of default risk heterogeneity and borrower selection in the U.S. mortgage market that elude existing quantitative frameworks in the literature. Finally, the model highlights scope for welfare improvements that arise from regulating mortgage credit on the basis of a loan's price rather than its DTI ratio.

Using loan-level data, I show that the correlation between DTI ratios and default risk changed significantly after the introduction of the current DTI limit in the conforming mortgage market. The policy requires mortgages to meet a DTI requirement at origination to be eligible for GSE purchase or guarantee unless the lender can substantiate the borrower's credit reputation. Pre-policy, higher DTI ratios are associated with lower credit scores and higher interest rates at origination. Post-policy, default risk exhibits a sharp discontinuity at the statutory DTI limit: borrowers with DTI ratios above it have higher credit scores and lower interest rates compared to borrowers just at or below the limit. The selection of more creditworthy borrowers into mortgages with larger DTI ratios in equilibrium is at odds with the predictions of a benchmark model of mortgage default. I also show that a fraction of borrowers at the limit are observationally no more likely to default than borrowers above it. This indicates that, conditional on having large DTI ratios, households nonetheless vary substantially in their default risk and suggests that some relatively "safe" borrowers may be constrained by the current DTI limit.

[1] A conforming loan is a mortgage that meets the underwriting standards of Freddie Mac and Fannie Mae, two government-sponsored enterprises (GSEs) active in the secondary mortgage market, and is thus eligible for GSE purchase or guarantee.

To rationalize these findings and conduct policy counterfactuals, I study an incomplete-markets life-cycle model with competitive loan pricing and long-term illiquid mortgages à la Kaplan, Mitman and Violante (2020) in which mortgage lenders relax a limit on DTI ratios at origination for households whose endogenous default probability is sufficiently low. This novel feature captures the institutional reality that lenders adopt more conservative underwriting for borrowers whose mortgages violate the statutory DTI requirement. Consistent with the structure of the secondary mortgage market, the government insures investors against credit losses from default by levying fees on lenders and a proportional income tax on households. Aggregate default in the model is costly because it imposes a deadweight loss on households through this channel.[2]

The calibrated model successfully matches observed heterogeneity in default risk conditional on DTI ratios. The underwriting technology yields borrower selection effects crucial for this result. Without it, the model predicts that the endogenous default risk of borrowers is monotonically increasing in their DTI ratio, and the variance of default risk across borrowers is counterfactually large. The underwriting technology generates an equilibrium in which only households with relatively low propensity to default hold mortgages with DTI ratios above the limit. However, it places no restrictions on the choices of households for whom the DTI limit is a hard constraint. DTI-constrained households who are most likely to default exit the mortgage market, lowering the dispersion in default risk across the population of borrowers in the stationary distribution. The remainder reduce their desired loan size until their DTI ratios fall just below the statutory limit. Thus, the model replicates the discontinuity in default risk at the DTI limit documented in the data.

By uncovering the counterfactual loan choices of households for whom the DTI limit binds, the structural model explains why households with large desired DTI ratios vary in their default probabilities. Households constrained by the limit along the extensive margin of debt are primarily renters who, in the absence of this constraint, would have financed a house purchase with a large loan. Their high default risk reflects the low home equity they would have conditional on becoming owners, leaving them vulnerable to adverse shocks to cash on hand that could make default in future states of the world optimal. Households constrained along the intensive margin of debt are mostly liquidity-constrained owners who refinance their loans for precautionary reasons. Their lower default risk reflects their substantial home equity, which gives them the option to liquidate their houses rather than default in response to future negative shocks.

A structural model consistent with the observed selection of borrowers into loans with

[2] It is generally accepted that GSE guarantees are underpriced and the government subsidizes credit risk in the mortgage market. See, for example, Congressional Budget Office (2018).

large DTI ratios is vital for studying the welfare effects of reforms to mortgage underwriting standards. To demonstrate this, I use the calibrated model to evaluate the aggregate and distributional effects of two proposed versions of the Dodd-Frank Act's ability-to-repay (ATR) rule relative to the current DTI limit. The ATR rule increases the cost of originating certain types of loans to lenders by increasing their legal liability in the event of borrower default. The original version of the rule penalized mortgages with a high DTI ratio at origination, whereas the revised rule penalizes loans with a high price. Compared to the current policy, the reform represents an effective relaxation of credit constraints because there are enough DTI-constrained households with sufficient ability to repay such that the pass-through of the legal cost to their equilibrium mortgage rate is limited. Nevertheless, because these households have characteristics that make them riskier than the existing population of borrowers, the aggregate foreclosure rate rises as such agents select into homeownership under the reform.

Theoretically, the welfare effects of the ATR rule are not *ex ante* obvious. Housing wealth has self-insurance value, but a higher default rate increases credit risk subsidies funded through taxation. Quantitatively, I find that aggregate welfare improves under both implementations of the ATR rule but by less under the original rule. More borrowers in the stationary distribution of the economy choose high-DTI than high-price loans, implying that the overall incidence of the ATR's legal costs—and, by extension, total credit losses from default that the government must insure in equilibrium—is greater. This finding highlights a broader policy trade-off between promoting homeownership and reducing mortgage default. Welfare gains are particularly large for agents who switch from renting to owning under the reform, but they are only a small fraction of the population. By contrast, the welfare loss from higher taxation experienced by a given household is generally small, but these losses are more widespread across the economy.

The paper proceeds as follows. In Section 2, I connect the paper to relevant literature. In Section 3, I present evidence on the variation of default risk with respect to DTI ratios. In Section 4, I develop a theoretical model that explains these facts. In Section 5, I explain the calibration strategy and validate model fit. In Section 6, I show that the calibrated model accounts for empirical patterns of loan selection and borrower heterogeneity. In Section 7, I use the model to study the Dodd-Frank ability-to-repay rule. Section 8 concludes.

2 Related literature

My paper builds on a growing body of macroeconomic research that features uninsurable idiosyncratic risk, long-term debt, and borrowing constraints in structural models of housing and mortgage markets.[3] My theoretical environment has much in common with models of household consumption that distinguish between liquid and illiquid assets.[4] With its inclusion of competitive loan pricing and endogenous default, my paper speaks to a related literature that uses equilibrium models to study bankruptcy and foreclosure.[5]

My empirical contribution is to document that an influential limit on debt payment-to-income ratios in the U.S. mortgage market is not applied uniformly across households and that relatively more creditworthy borrowers hold loans that violate this limit. Thus, in the part of the DTI ratio distribution where these these constraints are more likely to bind, a larger DTI ratio is not necessarily indicative of greater inability to repay. By contrast, the benchmark structural model used in macroeconomics to study mortgage default predicts that DTI ratios are increasing in the underlying probability of default. Consequently, my theoretical contribution is to embed a simple loan underwriting technology in the standard framework whereby the strictness of a DTI constraint at origination is a function of a borrower's endogenous probability of default and to show this can rationalize the observed relationship of default risk with respect to DTI ratios.[6]

My paper adds to ongoing studies of the effects of *ex ante* regulations in the U.S. mortgage market introduced after the Great Recession.[7] Because DTI ratios and default probabilities are equilibrium objects, my approach uses a calibrated structural model to conduct inference on unobservable household characteristics that rationalize documented patterns of loan choice and to conduct policy counterfactuals. My quantitative results reveal the rich heterogeneity that exists across households with similar desired DTI ratios and the importance of accounting for this when evaluating household leverage regulations. My paper also draws on studies of government guarantees in the U.S. mortgage market. Because households ultimately bear the cost of these guarantees, tighter borrowing constraints may improve welfare by reducing leverage and default rates.[8]

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[3]Davis and Van Nieuwerburgh (2014) and Piazzesi and Schneider (2016) provide overviews of this literature. For specific examples, see Beraja et al. (2019), Berger et al. (2017), Boar, Gorea and Midrigan (2022), Guren, Krishnamurthy and McQuade (2021), Guren et al. (2021), Kaplan, Mitman and Violante (2020), and Wong (2021).

[4]See Berger and Vavra (2015) and Kaplan and Violante (2014).

[5]See Chatterjee et al. (2007), Chatterjee and Eyigungor (2015), Chen, Michaux and Roussanov (2020), Garriga and Hedlund (2020), and Mitman (2016).

[6]Corbae and Quintin (2015) propose a theory in which borrowers choose between low- or high-down payment fixed-rate mortgages. Their model place strong restrictions on housing tenure and loan adjustment choices. Chambers, Garriga and Schlagenhauf (2009) study loan structure in general equilibrium but abstract from default. Gete and Zecchetto (2018) study an environment with different loan types; however, they model mortgages as one-period liquid debt and abstract from a life-cycle savings motive.

[7]See Bhutta and Ringo (2015), DeFusco, Johnson and Mondragon (2020), and Greenwald (2018).

[8]See Elenev, Landvoigt and Niewuerburgh (2016), Gete and Zecchetto (2018), and Jeske, Kreuger and Mitman (2013).

3 Empirical evidence on DTI ratios and default risk

Although the conforming mortgage market has operated under a DTI limit of 45% since 2009, it is not necessarily a hard constraint. I exploit differences between loans on either side of the limit to establish that, under this policy, a higher DTI ratio need not imply a lower ability to repay. Before 2009, default risk is strictly increasing in the DTI ratio at origination. Afterwards, a discontinuity emerges at the 45% cutoff: compared to borrowers at the limit, those above it have significantly higher credit scores and face lower costs of borrowing in equilibrium. This implies that, among households who are most likely to be affected by the DTI limit, there is substantial variation in default risk even holding DTI ratios fixed. Furthermore, I find that some borrowers with DTI ratios at the limit are observationally no more likely to default than borrowers with DTI ratios above the limit. This suggests that some relatively creditworthy households may be constrained by the DTI limit.

3.1 Institutional background on DTI limits and data description

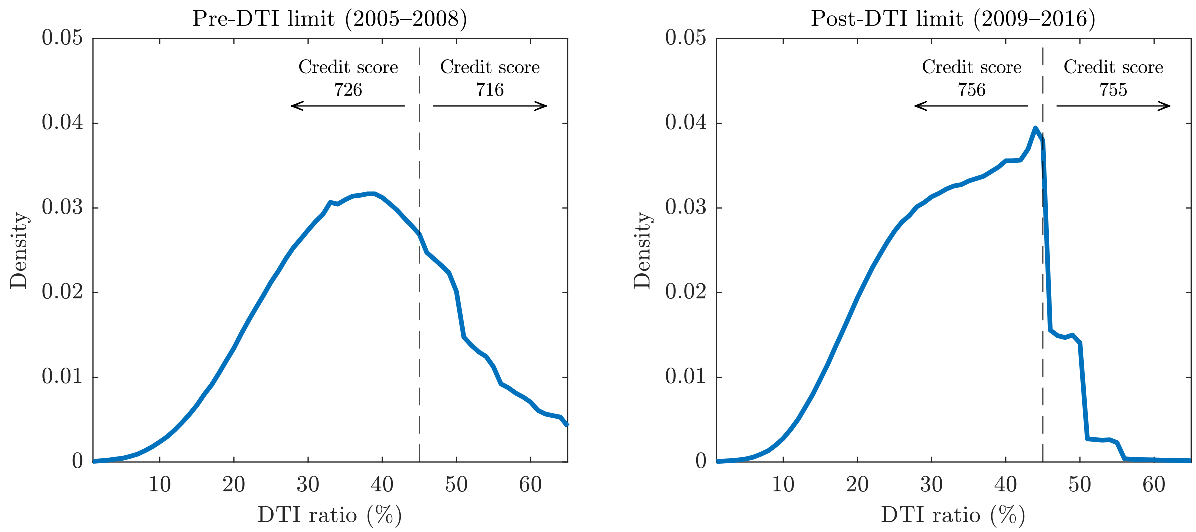
The empirical analysis concerns a reform to the underwriting standards of Freddie Mac, one of two large government-sponsored enterprises that purchase mortgage originations from primary lenders and issue securities backed by those mortgages to investors on the secondary market. The GSEs guarantee investors against credit losses from default by charging lenders a fee. They only purchase or guarantee mortgages that meet their underwriting standards in order to reduce exposure to credit losses.

In March 2009, Freddie Mac introduced a requirement that limited the back-end debt payment-to-income ratio of borrowers at origination to 45%.[9] Lenders have some discretion to originate a mortgage with a DTI ratio above the limit at the cost of applying more strict underwriting standards. These could require the borrower to possess enough liquid assets to constitute an ability to repay regardless of income; make a down payment of at least 25%; or have a strong credit score, defined as 740 or higher, combined with the lender's written assurance that "the borrower's credit reputation is excellent." However, there are no sufficient conditions for ensuring that Freddie Mac would purchase a mortgage with a DTI ratio exceeding the requirement.

I use the Freddie Mac Single Family Loan-Level Dataset for the empirical analysis because it represents the segment of the mortgage market most directly affected by this policy. The dataset contains quarterly loan-level data on fully amortized fixed-rate mortgage originations with full documentation that have been purchased or guaranteed by Freddie Mac since 1999. I focus on loans that were originated between 2005 and 2016 with 30-year terms; collateralized

[9] See https://guide.freddiemac.com/app/guide/section/5401.2.

FIGURE 1. DTI ratios and credit scores before and after the 45% DTI limit



Notes: Mortgages are grouped into 1-percentage point bins. The dashed black vertical line indicates the 45% DTI limit. The mean credit score that appears to the left (right) of the dashed black line is calculated from mortgages originated with a DTI ratio less (greater) than 45%. Source: Freddie Mac Single Family Loan-Level Dataset.

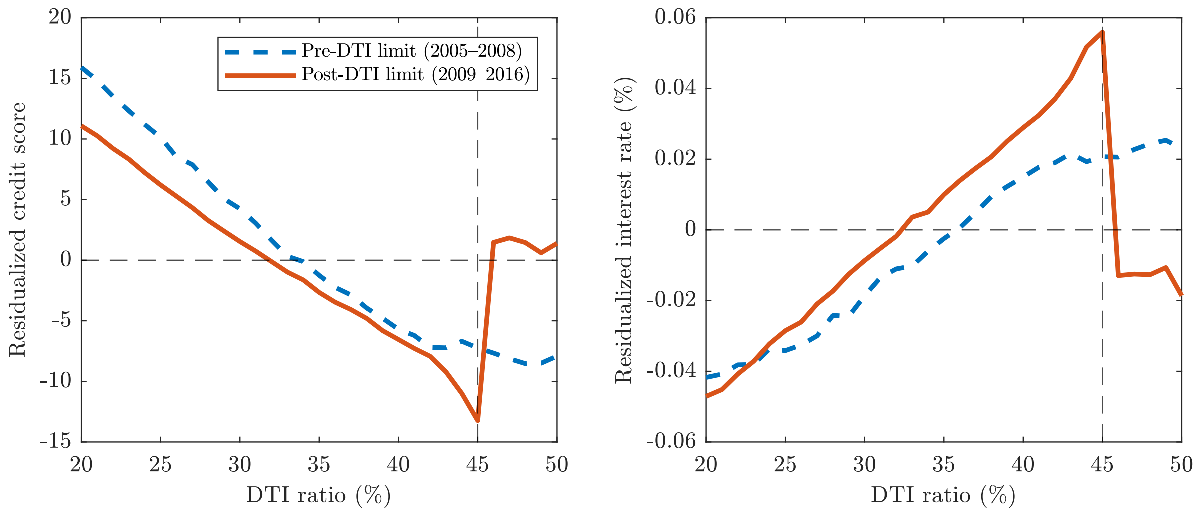
by owner-occupied housing; and have non-missing DTI ratios, credit scores, interest rates, and loan-to-value (LTV) ratios.[10] I will refer to loans that do and do not meet a statutory DTI limit as *low-DTI loans* and *high-DTI loans*, respectively, for the rest of the paper.

3.2 Selection of creditworthy borrowers into high-DTI loans

Although there has been a large reduction in the share of mortgages with DTI ratios above 45% in the post-policy period, Figure 1 shows that these loans still account for roughly 9% of originations. Accompanying this is a stark change in the distribution of credit scores at origination—used here as an observable proxy for the borrower's underlying default risk—across DTI ratios. In the pre-policy period, high-DTI borrowers had a mean credit score at origination of 716, compared to 726 for low-DTI borrowers. This observation underlies common justifications for using DTI limits as a policy instrument for reducing household leverage and mortgage default. While the average credit score of borrowers obtaining new loans has risen since the Great Recession, high-DTI borrowers have become more creditworthy relative to their pre-policy level than low-DTI borrowers. Mean credit scores on either side of the limit have been nearly identical since 2009, suggesting that DTI ratios have become less correlated with default risk.

[10]The resulting sample has around 10 million loans. See Table 8 in Appendix A.1 for summary statistics.

FIGURE 2. Credit scores and interest rates vs. DTI ratios at origination



Notes: The credit score and interest rate are each residualized with respect to a vector of year-quarter dummy variables. Averages are computed for each 1-percentage point DTI bin for pre- and post-2009Q1 observations separately. The vertical dashed black line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.

Figure 2 confirms this intuition by showing that, under current regulations, borrower default risk is non-monotonic in DTI ratios in the neighborhood of the statutory limit. In the left-hand panel, I plot credit scores, residualized with respect to an aggregate time trend, as a function of DTI ratios for loans originated before and after the implementation of the DTI limit in 2009.[11] Before 2009, a borrower's credit score was monotonically decreasing in their DTI ratio. From 2009 onward, a clear discontinuity in credit scores emerges at the statutory DTI limit as borrowers above the limit become more creditworthy while borrowers immediately below it become less creditworthy. One way to contextualize the relative improvement in the ability to repay of high-DTI borrowers is to note that borrowers with a DTI ratio of 50% have credit scores comparable to those of borrowers with a DTI ratio of 30%. To the left of the limit, though, credit scores remain strictly decreasing in the DTI ratio; borrowers with DTI ratios exactly equal to 45% have the lowest credit scores of all. The observed non-monotonicity of credit scores with respect to DTI ratios in the data suggests that, in equilibrium, lenders only exercise the discretion they have to originate high-DTI loans in favor of borrows whom they judge to be sufficiently creditworthy.

The right-hand panel of Figure 2 displays how borrowers' interest rates—another observable proxy of their default risk—vary with respect to their DTI ratios pre- and post-

[11] Year-quarter fixed effects control for aggregate changes in loan underwriting standards and interest rates. As shown in Figure 8 in Appendix A.1, the average credit score and interest rate at origination have increased and decreased, respectively, over the sample period.

policy. Before 2009, interest rates are increasing in DTI ratios through the 45% cutoff. After 2009, they becomes discontinuous at this point: borrowers to the right of the DTI limit receive below-average interest rates, while those immediately to the left receive above-average interest rates. This finding is especially striking because a high interest can mechanically lead to a large DTI ratio, all else equal.[12] The association of high-DTI loans with relatively *lower* interest rates suggests important differences in the composition of borrowers across the 45% DTI limit.

In Appendix A.2, I estimate a difference-in-differences regression to control for other observable loan-level characteristics. The differences in credit scores and interest rates around the 45% DTI limit in Figure 2 remain statistically significant at the 1% level after holding these characteristics fixed. The regression estimates imply that, relative to low-DTI loans, the credit score and interest rate on high-DTI loans increases by 10.7 and decreases by 5.5 basis points, respectively, in the post-policy period.

3.3 Heterogeneity in default risk among borrowers on the DTI limit

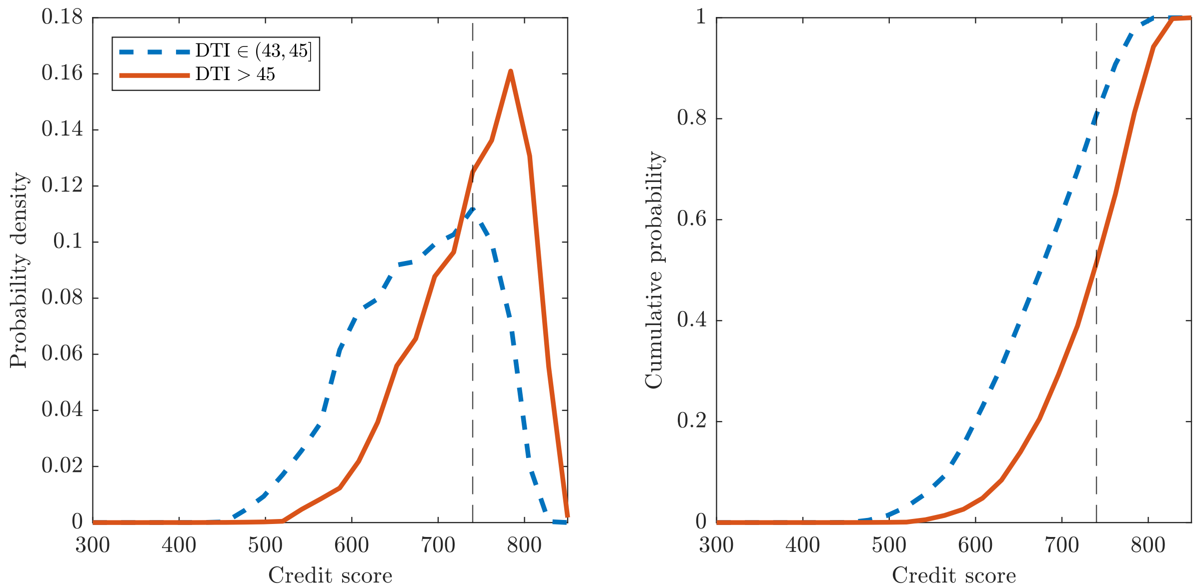
Next, I document that, despite the fact that borrowers with DTI ratios at the statutory maximum have the lowest credit scores on average in the sample, they are not uniform in their ability to repay. In Figure 3, I plot distributions of credit scores associated with mortgages originated in the post-policy period separately for borrowers with DTI ratios above 45% versus those with DTI ratios between 43% and 45%.[13] It reveals the existence of significant dispersion in the credit scores among borrowers found at the limit. The credit scores of high-DTI borrowers display lower variance. This is expected, given lenders can only qualify relatively safe borrowers for high-DTI loans.

Of particular note is the overlap in the distribution of credit scores for these two groups of borrowers. This suggests that some borrowers at the DTI limit may be constrained by it despite not being observationally more likely to default than borrowers for whom the limit was relaxed. One indication of this is the share of borrowers at the limit with credit scores of 740 or above. Since 2009, around 20% of mortgage originations at the DTI limit have gone to borrowers with credit scores in this range. This cutoff is relevant because, as mentioned in Section 3.1, it is one possible condition under which a lender may waive the DTI requirement, though the data make clear that a credit score in this range is neither necessary nor sufficient in qualifying for a high-DTI loan.

[12] A large DTI ratio could also reflect a large initial loan size. In Appendix A.1, I verify that DTI ratios above the 45% statutory requirement are not driven by this. Indeed, high-DTI loans have been associated with *lower* LTV ratios, relative to loans just below the limit, since 2009.

[13] A DTI threshold of 43% is relevant in the policy counterfactual in Section 7. Restricting the comparison to borrowers who have a DTI ratio equal to 45% results produces nearly identical results.

FIGURE 3. Heterogeneity in credit scores across borrowers conditional on observed DTI ratio



Notes: The distributions are constructed by sorting mortgages originated in 2009Q1 and later into 26 credit score bins. The dashed black vertical line marks the credit score of 740. Source: Freddie Mac Single Family Loan-Level Dataset.

3.4 Robustness and discussion of empirical results

In Appendix A.3, I show that the empirical findings are robust to using a dataset on mortgages bought or guaranteed by Fannie Mae despite the fact that it had a looser DTI limit than Freddie Mac over the sample period. This provides evidence that the 45% DTI limit effectively applied to the conforming mortgage market as a whole during the sample period. There may be concerns that borrowers constrained by the DTI limit in the conforming mortgage market could avoid this requirement by choosing a non-conforming loan instead. Empirical findings in the literature, however, suggest that substitution along this margin was limited. For instance, Bhutta and Ringo (2021) find evidence consistent with binding DTI caps at 43% and 45% among purchase loans insured by the Federal Housing Administration. Administrative data from the Home Mortgage Disclosure Act also reveal the influence of the 45% DTI limit among different segments of the non-conforming mortgage market.[14]

There may also be concerns that, by focusing on fixed-rate mortgages with 30-years terms, the empirical analysis may not account for the effect of loan maturity on DTI ratios. In Appendix A.4, I show that the stylized facts are robust to including Freddie Mac mortgages

[14]See https://files.consumerfinance.gov/f/documents/cfpb_data-points_updated-review-hmda_report.pdf. The Home Mortgage Disclosure Act only required lenders to report DTI ratios beginning in 2018, making that dataset unsuitable for studying the effects of a policy introduced in 2009.

with non-30-year terms in my sample. Almost all such mortgages are refinance loans with maturities shorter than 30 years. They are associated with smaller loan amounts, lower interest rates, and higher credit scores. They are also much less likely to be high-DTI loans. This suggests that non-30-year fixed-rate mortgages should be relatively unaffected by the policies studied in this paper.

4 A model with mortgage default and flexible borrowing limits

Motivated by empirical evidence that the debt payment-to-income limit is a soft constraint for sufficiently creditworthy borrowers, I study an incomplete-markets life-cycle model with a competitive mortgage market augmented with a novel loan underwriting technology. In this framework, a household's endogenous probability of default must be sufficiently low for their loan chocie to be exempt from a DTI requirement. This tractably models the ability of lenders to originate high-DTI loans in exchange for more conservative underwriting and generates a stationary distribution in which some—but not all—households can violate an ad hoc borrowing constraint.

To incorporate salient features of the U.S. secondary mortgage market, the model also has a credit risk subsidy à la Gete and Zecchetto (2018) in which investors that provide funds to mortgage lenders are guaranteed against losses from default. To cover these expenses, the government charges lenders an up-front fee on each mortgage origination and levies a proportional tax on households' labor income. This captures the fact that the government implicitly subsidizes credit risk in the mortgage market.

4.1 Model overview

Time is discrete. There is a constant population of overlapping generations of households who split their lives between working and retirement. Working households receive an age-specific endowment income subject to uninsurable idiosyncratic risk. Retired households receive a constant pension income. Households can save in a liquid asset and illiquid housing wealth. Homeowners can borrow through illiquid long-term, fixed-rate mortgage contracts subject to borrowing limits at origination. Debtors have the option to default. Households derive utility each period from nondurable consumption and housing services. They can either rent housing services or purchase a house that yields a service flow each period. House prices are subject to uninsurable idiosyncratic shocks. Owner-occupied housing is also subject to depreciation shocks.

A continuum of competitive, risk neutral, and infinitely lived financial intermediaries store the liquid savings of and supply mortgage debt to households. Because financial intermediaries observe the household's idiosyncratic state, the equilibrium interest rate on a newly originated mortgage reflects the household's endogenous default risk and is such that the lender makes zero expected profit on a loan-by-loan basis.

Preferences and endowments

A household maximizes expected discounted lifetime utility,

$$\max \ \mathbb{E}\left\{\sum_{j=1}^{T}\left[\beta^{j-1}u_j\left(c_j,s_j\right)\right]+\beta^{T}\nu\left(W_{T+1}\right)\right\},$$

where j indexes the household's age and $\beta \in (0,1)$ is the subjective discount factor. The flow utility function $u(\cdot)$ satisfies the Inada conditions and is given by

$$u_j\left(c_j,s_j\right)=\frac{1}{1-\sigma}\left[\alpha_j c_j^{1-\vartheta}+\left(1-\alpha_j\right)s_j^{1-\vartheta}\right]^{\frac{1-\sigma}{1-\vartheta}},$$

where c is nondurable consumption and s is housing services. $\sigma > 0$ is the coefficient of relative risk aversion, and $1/\vartheta > 0$ is the elasticity of substitution between nondurable consumption and housing services. $\alpha_j \in (0,1)$ is the preference weight on nondurable consumption that potential depends on age in order to account for life-cycle components of demand for housing services that are not explicitly modeled. Households have a bequest motive whereby they receive expected discounted utility from end-of-life wealth W_{T+1} according to

$$\nu\left(W_{T+1}\right)=B\mathbb{E}\left\{\frac{W_{T+1}^{1-\sigma}}{1-\sigma}\right\}.$$

The parameter $B > 0$ controls the strength of the bequest motive.

A household supplies labor inelastically from age 1 until retirement at age T_R. While working, a household's log income is the sum of a deterministic component that is a function of age χ_j and an idiosyncratic component z that evolves according to the first order Markov process

$$z'=\rho_z z+\varepsilon',\ \varepsilon' \overset{i.i.d.}{\sim} \mathcal{N}\left(0,\sigma_\varepsilon^2\right).$$

A retired household receives a constant pension that is a deterministic function $\Phi(\cdot)$ of earnings at age $T_R - 1$. Thus, the process for log income is

$$\log y_j\left(z\right)=\begin{cases}\chi_j+z & \text{if } 1\leq j\leq T_R-1\\ \Phi\left(y_{T_R-1}\left(z\right)\right) & \text{if } T_R\leq j\leq T.\end{cases}$$

Households face a proportional tax $\tau \in (0,1)$ on their labor income that they take as given. This tax rate captures the extent to which credit risk guarantees in the secondary mortgage market are underpriced.[15]

Liquid savings

Households can save in a one-period liquid asset a, subject to a no-borrowing constraint, in the form of deposits held by the financial intermediaries. Intermediaries have access to international capital markets where the net supply of safe assets determines the risk-free rate $r > 0$. A zero-profit condition implies households also earn return r on liquid assets.

Housing

Households obtain housing services through the rental or owner-occupied housing market. If an agent buys a house, they chose a house of size h from a discrete grid at price p and receive a service flow $s = h$ each period. The discreteness grid captures the indivisibility of housing as an asset class. The supply of houses of all sizes is perfectly elastic. Following Berger et al. (2017) and Mitman (2016), house prices are subject to an idiosyncratic shock and follow the first order Markov process

$$\log p' = \rho_p \log p + \eta', \ \eta' \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_\eta^2\right).$$

Households who adjust their house size pay a fixed transaction cost $\kappa_h > 0$. As in Chatterjee and Eyigungor (2015), owner-occupied housing is subject to an i.i.d. depreciation shock $\delta \in (0,1)$, where

$$\delta = \begin{cases} \bar{\delta} & \text{with probability } \zeta \\ \underline{\delta} & \text{with probability } 1-\zeta \end{cases}$$

and $\bar{\delta} > \underline{\delta}$, so that owners pay a housing maintenance cost of δph each period.

Otherwise, a household can rent s units of housing services at a rate Rp per unit each period, where $R > 0$ denotes the exogenous rent-price ratio. The supply of rental housing is perfectly elastic. Adjusting the quantity of rental housing between periods does not incur a transaction cost, and renters do not experience depreciation shocks.

It should be noted that the model abstracts from endogenous house prices. As shown in Kaplan, Mitman and Violante (2020), the assumptions of long-term mortgage contracts and no segmentation between owner-occupied and rental housing markets jointly imply that

[15] For example, in a study of options for housing finance reform, the Congressional Budget Office (2018) projected that "the GSEs will guarantee almost \$12 trillion in new [mortgage-backed securities] over the next 10 years and that those guarantees will cost the government about \$19 billion."

changes in credit conditions—such as the DTI limits studied here—have minimal effect on house prices in equilibrium. Because the model shares these two features, incorporating feedback effects through house prices should not alter the results substantially. Within the macro-housing literature, the pass-through of credit conditions to house prices remains an open quantitative question that is outside of the scope of this paper to address.[16]

Mortgages

Owners can use their house as collateral for fixed-rate mortgage debt m. Loan size at origination must satisfy constraints on loan-to-value and debt payment-to-income ratios. Households borrow at an equilibrium interest rate r_m that depends on their idiosyncratic state at origination and stays constant for the remaining duration of the loan. Thus, the current state of an age-j household is given by $\omega \equiv (a, h, \delta, p, m, r_m, z)$.

The contribution of the model is the introduction of a default risk threshold that borrowers must satisfy in equilibrium for a high-DTI loan to be feasible. This gives rise to "flexible" DTI constraint that some households can violate. As discussed in Section 3.1, GSE underwriting guidelines do not provide necessary or sufficient conditions for waiving the DTI limit. The default risk threshold can be thought of as reduced-form representation of a lender's discretion to originate a high-DTI mortgage if they are assured of the borrower's ability to repay. As will be shown later in the quantitative results, this class of models without this feature counterfactually predicts that households with higher probability of default have larger DTI ratios in equilibrium.

The DTI limit states that the minimum mortgage payment $\pi_{min,j}(m, r_m)$ must be less than a fraction $\lambda_j(\omega)$ of the household's income $y_j(z)$, i.e.,

$$\pi_{min,j}(m, r_m) \leq \lambda_j(\omega)\, y_j(z),$$

where

$$\lambda_j(\omega) = \begin{cases} \lambda - \varsigma_j(\omega) & \text{if } \psi_j(\omega) > \Psi \\ \infty & \text{if } \psi_j(\omega) \leq \Psi. \end{cases} \tag{1}$$

If the household's endogenous default probability $\psi_j(\omega)$ is greater than a threshold value $\Psi \in [0, 1]$, then their DTI ratio at origination must be less than $\lambda - \varsigma_j(\omega)$, where $\lambda \in (0, 1)$ is the statutory DTI requirement and $\varsigma_j(\omega)$ is an exogenous offset term that captures non-

[16] See Greenwald and Guren (2022) and references therein.

mortgage debt service and housing expenses relative to income.[17] If their endogenous default probability meets the threshold, then the DTI constraint is fully relaxed.

The other features of mortgage debt are common to the literature. A LTV constraint

$$m \leq \theta ph$$

limits the loan size to a fraction $\theta \in (0, 1)$ of the value of the home ph. A household who obtains a new loan pays a fixed transaction cost $\kappa_m > 0$ and can only hold one loan at a time. The loan balance is amortized over the remaining lifetime of the household, implying that the minimum loan payment due each period is

$$\pi_{min,j}(m, r_m) = \frac{(1 + r_m)^{T-(j-1)}}{(1 + r_m)^{T-(j-1)} - 1} r_m m. \tag{2}$$

4.2 Household's optimization problem

The household's optimization problem is written recursively. The expected discounted lifetime utility of an age-j renter in state ω is

$$W_j^R(\omega) = \max_{x_j(\omega)} \left\{ V_j^R(\omega), V_j^M(\omega) \right\}. \tag{3}$$

The expected discounted lifetime utility of an age-j homeowner in state ω is

$$W_j^O(\omega) = \max_{x_j(\omega)} \left\{ V_j^R(\omega), V_j^M(\omega), V_j^P(\omega), V_j^D(\omega) \right\}. \tag{4}$$

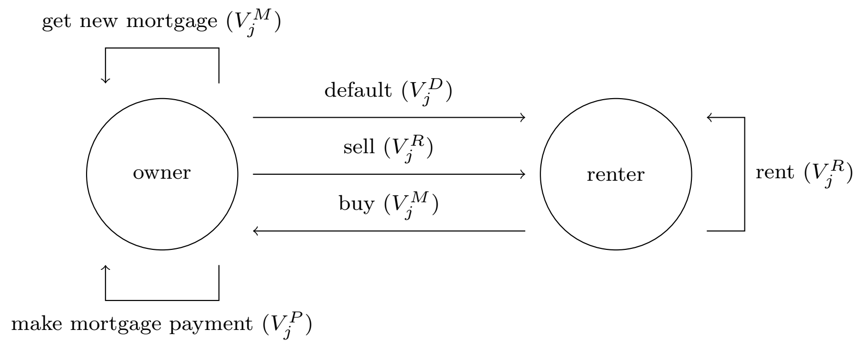
The value functions inside the maximum operators of Equations (3) and (4) correspond to discrete choices over housing tenure and loan adjustment available in state ω. $V_j^R(\omega)$ is the value of renting; $V_j^M(\omega)$ is the value of obtaining a new loan; $V_j^P(\omega)$ is the value of making a payment on an existing loan; and $V_j^D(\omega)$ is the value of defaulting on outstanding debt.[18] $x_j(\omega) \in \{R, M, P, D\}$ denotes the decision to rent, obtain a new loan, continue with an existing loan, and default, respectively. Figure 4 summarizes these choices and flows between renting and owning.

The timing of the model is as follows. At the beginning of the period, idiosyncratic shocks

[17] See Equation (7) for an expression for $\psi_j(\omega)$ after the relevant value functions have been defined. I follow the literature—e.g., Boar, Gorea and Midrigan (2022) and Greenwald (2018)—in adjusting the statutory DTI limit downward because underwriting standards target a borrower's back-end DTI ratio and non-mortgage liabilities are not otherwise captured by the model. GSE underwriting guidelines include housing expenses like mortgage insurance premiums, real estate taxes, etc. when calculating a household's DTI ratio.

[18] Owners only have the option to default conditional on having strictly positive debt. I account for this in the numerical solution of the model by setting $V_j^D(\omega)$ to a large negative number in states where $m = 0$.

Figure 4. Discrete choices over housing tenure and loan adjustment in the household's problem



Notes: "Owner" and "renter" refer to an agent's predetermined housing tenure.

are realized. Households then solve the optimization problems associated with available housing tenure and loan adjustment choices and select the option that yields the highest expected lifetime utility. Consumption occurs at the end of the period.

If renting, a household solves

$$
\begin{aligned}
V_j^R(\omega) = \max_{c,s,a'} \; & u_j(c,s) + \beta \mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^R(\omega') \\
& \text{s.t.} \\
c + Rps + a' & \le (1-\tau) y_j(z) + (1+r) a + (1-\delta) ph - (1+r_m) m - \mathbb{1}_{h \neq 0} \kappa_h \\
a' & \ge 0 \\
\omega' & = (a', 0, \delta', p', 0, 0, z').
\end{aligned}
\tag{5}
$$

If owning and obtaining a new mortgage, a household solves

$$
\begin{aligned}
& V_j^M(\omega) = \max_{c,a',h',m'} \; u_j(c,h') + \beta \mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^O(\omega') \\
& \quad \text{s.t.} \\
& c + a' + ph' \le (1-\tau) y_j(z) + (1+r) a + (1-\delta) ph - (1+r_m) m + m' - \mathbb{1}_{h' \neq h} \kappa_h - \mathbb{1}_{m'>0} \kappa_m \\
& m' \le \theta p h' \\
& \pi_{min,j}\left(m', r'_{m,j}(\omega)\right) \le \lambda_j(\omega) y_j(z) \\
& a' \ge 0 \\
& \omega' = \left(a', h', \delta', p', m', r'_{m,j}(\omega), z'\right),
\end{aligned}
\tag{6}
$$

where Equation (1) defines the tightness of the DTI constraint, $\lambda_j(\omega)$. The household's endogenous default risk determines the feasibility of a high-DTI loan and is computed as

$$\psi_j(\omega) \equiv \mathbb{E}_{\delta',p',z'|p,z}\left\{\mathbb{1}_{x_{j+1}(\omega')=D|x_j(\omega)=M}\right\}, \tag{7}$$

i.e., the probability that an age-j household defaults at age $j+1$ conditional on getting a new loan in their current state ω. This problem makes explicit a household's interaction with the mortgage market through their cost of borrowing. The interest rate $r'_{m,j}(\omega)$ is determined in equilibrium, affects the size of the minimum mortgage payment, and appears in tomorrow's state ω' because it is fixed for the duration of the loan.

If continuing with an existing loan, an owner solves

$$\begin{aligned}
V_j^P(\omega) = \max_{c,a',m'}\ & u_j(c,h) + \beta\mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^O(\omega') \\
\text{s.t.}\ & \\
& c + \delta ph + a' \leq (1-\tau)\, y_j(z) + (1+r)\, a - (1+r_m)\, m + m' \\
& m' \leq (1+r_m)\, m - \pi_{min,j}(m, r_m) \\
& a' \geq 0 \\
& \omega' = (a', h, \delta', p', m', r_m, z').
\end{aligned} \tag{8}$$

In this case, the household's state in the next period reflects the fact that they continue with their predetermined house size and mortgage interest rate.

If defaulting, an owner with outstanding debt solves

$$\begin{aligned}
V_j^D(\omega) = \max_{c,s,a'}\ & u_j(c,s) - \xi + \beta\mathbb{E}_{\delta',p',z'|p,z}\left[\varphi W_{j+1}^R(\omega') + (1-\varphi)\, V_{j+1}^R(\omega')\right] \\
\text{s.t.}\ & \\
& c + Rps + a' \leq (1-\tau)\, y_j(z) + (1+r)\, a \\
& a' \geq 0 \\
& \omega' = (a', 0, \delta', p', 0, 0, z').
\end{aligned} \tag{9}$$

Because a defaulting household discharges existing debt and loses their house, they do not cover the minimum mortgage payment or housing maintenance costs due that period. They incur a flow utility loss $\xi > 0$ and are excluded from owning a house and borrowing for a stochastic period of time. The parameter $\varphi \in (0,1)$ is the probability that the household regains access to those markets in the next period.

At age T, the household must repay all debt and cannot get a new loan. This imposes the restriction that $m' = 0$ on the optimization problems associated with obtaining a new loan and continuing with an existing mortgage. End-of-life wealth consists of liquid savings and, if applicable, the value of the home net of depreciation, $W_{T+1} = (1+r)\, a' + (1-\delta')\, p'h'$.

4.3 Financial intermediary's optimization problem

A continuum of financial intermediaries maximize expected profit and issue mortgage contracts. Lenders are risk neutral, infinitely lived, and perfectly competitive. They have complete information about a household's current state and decision rules.[19] They discount the future at rate $r + \phi$, where the parameter $\phi > 0$ captures mortgage servicing costs. Consistent with the reliance of primary lenders on securitization for funding, I assume that the intermediaries are owned by deep-pocked investors who receive cashflows from outstanding mortgage loans. Lenders pay a guarantee fee equal to a fraction $g \in (0,1)$ of the face value of each loan they originate to the government to cover investors' credit losses from default.

The present value of an outstanding mortgage held by an age-j household in state ω is

$$\Pi_j(\omega) = \begin{cases} (1+r_m)\, m & \text{if repay} \\ (1-\delta)\, ph - \gamma & \text{if default} \\ (1+r_m)\, m - m_j'(\omega) + \dfrac{1}{1+r+\phi} \mathbb{E}_{\delta',p',z'|p,z} \Pi_{j+1}(\omega') & \text{otherwise,} \end{cases} \tag{10}$$

where $\omega' = \left(a_j'(\omega), h_j'(\omega), \delta', p', m_j'(\omega), r_{m,j}'(\omega), z'\right)$. If the borrower pays off their loan, then the lender receives the remaining balance plus interest. If the borrower defaults, then the lender recovers the value of the house posted as collateral net of depreciation and an exogenous foreclosure cost $\gamma > 0$.[20] If the borrower continues with the loan, then the lender receives their mortgage payment plus the continuation value of the loan.

On a loan-by-loan basis, the face value of a newly originated loan equals the expected present value of its future cash flows in equilibrium net of the guarantee fee. Thus, the lender offers an interest rate $r_{m,j}'(\omega)$ that satisfies the zero-expected profit condition

$$(1+g)\, m_j'(\omega) = \frac{1}{1+r+\phi} \mathbb{E}_{\delta',p',z'|p,z} \Pi_{j+1}(\omega') . \tag{11}$$

Note that, without the additional friction introduced by the underwriting technology, lenders supply enough credit to meet the mortgage demand of an age-j household in state ω

[19] This assumption is consistent with the general approach of the quantitative macro-housing literature. Exceptions include Chatterjee et al. (2022) and Guler (2015).

[20] The foreclosure cost parameter stands in for legal costs associated with judicial foreclosures, administrative overhead, and additional lack of maintenance that results from the property remaining unoccupied.

regardless of their DTI ratio at origination provided that Equation (11) holds.

4.4 Government

A government insures investors in the mortgage market against credit losses, collects guarantee fees from lenders, and levies a proportional labor income tax on households. The tax rate τ is adjusted residually so that so that the budget constraint for mortgage guarantees,

$$\begin{aligned}\sum_{j=1}^{T}\left[\int \mathcal{L}_j(\omega)\,\mathbb{1}_{x_j(\omega)=D}\Lambda_j(d\omega)\right] &= g\sum_{j=1}^{T}\left[\int m'_j(\omega)\,\mathbb{1}_{x_j(\omega)=M}\Lambda_j(d\omega)\right]\\ &+\tau\sum_{j=1}^{T}\left[\int y_j(z)\,\Lambda_j(d\omega)\right],\end{aligned} \tag{12}$$

holds in equilibrium. $\Lambda_j(d\omega)$ is the distribution of age-j households over states, and the credit loss from default by an age-j household in state ω is

$$\mathcal{L}_j(\omega) \equiv (1+r_m)\,m - [(1-\delta)\,ph - \gamma]\,. \tag{13}$$

4.5 Equilibrium

I solve for the recursive stationary equilibrium of the model. Appendix B.1 defines the equilibrium, and Appendix C describes the numerical solution algorithm in detail.

5 Calibration

The goal of the calibration is to ensure that the stationary distribution of households is consistent not only with household income and balance sheets but also mortgage market outcomes under the debt payment-to-income limit documented in Section 3. This makes model a reasonable representation of the institutional status quo and an appropriate setting for policy evaluation. Where possible, I assign parameter values directly by relying on external empirical evidence or standard values in the literature. I calibrate the remaining parameters internally. I discuss the two sets of parameters separately, then show that the calibrated model does well in matching life-cycle profiles of household balance sheets and consumption, along with aggregate moments regarding homeownership, mortgage debt, and default.

5.1 Externally calibrated parameters

Table 1 lists the directly assigned parameters and their sources.

Table 1. Externally assigned parameters

	Description	Value	Source
χ_j	Age-specific income		PSID (1999–2017)
g	Guarantee fee (bp)	16	FHFA
μ_1	Distribution of age-1 households		SCF (2016)
Φ	Pension income		Guvenen and Smith (2014)
ς_j	DTI constraint offset		SCF (2016) and AHS (2017)
$\underline{\delta}$	Low depreciation rate (%)	1.064	PSID (2005–2017)
λ	Statutory DTI limit (%)	45	Freddie Mac
φ	Exclusion from mortgage market	0.143	Experian
r	Risk-free rate (%)	1.270	FRED (1971–2016)
ρ_p	Persistence of house price shock	0.970	Mitman (2016)
ρ_z	Persistence of income shock	0.977	Storesletten, Telmer and Yaron (2004)
σ	Risk aversion	2	Standard in literature
σ_ε	Std. dev. of income shock	0.155	Storesletten, Telmer and Yaron (2004)
σ_η	Std. dev. of house price shock	0.080	Mitman (2016)
T	Number of model periods	56	U.S. life expectancy of age 80
T_R	Retirement age	41	Retirement at age 65
θ	Statutory LTV limit (%)	85	Greenwald (2018)
ϑ	Inv. elasticity of sub. btw. c and s	0.800	Piazzesi, Schneider and Tuzel (2007)

Preferences and endowments

One period in the model is one year. Households enter the model at age 25, retire at age 65, and die at age 80, implying $T = 56$ and $T_R = 41$. Following Piazzesi, Schneider and Tuzel (2007), I set ϑ to match an elasticity of substitution between nondurable consumption and housing services of 1.25. The coefficient of relative risk aversion σ is 2, a standard value in the macroeconomics literature.

Following Storesletten, Telmer and Yaron (2004), I set the persistence of shocks to the idiosyncratic component of income ρ_z to 0.977 and their standard deviation σ_ε to 0.155. Following Kaplan and Violante (2014), I use data from the 1999–2017 waves of the Panel Study of Income Dynamics (PSID) to estimate the deterministic age-dependent component of income. I parameterize pension income using the procedure described by Guvenen and Smith (2014). Details on the estimation of the income process are in Appendix B.2.

Liquid savings

The risk-free rate r is set to 1.27%, the difference between the 1-year Treasury constant maturity rate and annual CPI inflation averaged over 1971–2016.

Housing

Following Mitman (2016), the persistence of the idiosyncratic house price shock ρ_p is 0.970 and its standard deviation σ_η is 0.080. The low depreciation rate $\underline{\delta}$ is 1.06% to match the mean ratio of annual home maintenance expenditures to home value in the PSID.[21]

Mortgages

The statutory DTI limit λ is 45%. I parameterize the DTI offset $\varsigma_j(\omega)$ by regressing household balance sheet and demographic variables on non-mortgage DTI ratios computed from the 2016 Survey of Consumer Finances (SCF). I use the estimated coefficients to project non-mortgage DTI ratios for agents in the model conditional their age and current state. Separately, I use the 2017 American Housing Survey to calculate the mean ratio of housing expenses relative to income. A household's DTI offset is the sum of their projected non-mortgage DTI ratio and the mean housing expense-to-income ratio. Further details are provided in Appendix B.3.

The probability of mortgage market re-entry after default φ is 0.143. This matches the 7-year period for which a foreclosure flag remains on a consumer's credit report. Following Greenwald (2018), the statutory LTV limit θ is 85%.[22] The guarantee fee g is 16 basis points, the average value of the upfront portion of the guarantee fee charged by the GSEs in 2016.[23]

Distribution of age-1 households

To initialize the stationary distribution of households, I stratify a sample of households with heads between ages 23–27 from the 2016 SCF into N_z groups according to their incomes to match the invariant distribution of the income shock z, where N_z is the number of discretized income states in the numerical solution of the model. For households in each z, I calculate the homeownership rate, the fraction of homeowners with mortgages, mean liquid assets, mean home equity, and the mean interest rate on the first mortgage. I then distribute households across states to match these balance sheet statistics from the SCF, as well as the invariant distributions of the depreciation and house price shocks.

[21] Specifically, I use question F87, "How much did you (and your family living there) spend altogether in [previous year] on home repairs and maintenance, including materials plus any costs for hiring a professional?"

[22] This captures the fact that a small fraction of conforming mortgages are originated with LTV ratios above the usual statutory requirement of 80%. However, as Greenwald (2018) shows, the distribution of and institutional requirements regarding LTV ratios in the conforming mortgage market was relatively unchanged during the time period I study. To keep the focus on the model on the effects of DTI limits, then, I assume a single hard constraint on LTV ratios.

[23] See https://www.fhfa.gov/AboutUs/Reports/Pages/Fannie-Mae-and-Freddie-Mac-Single-Family-Guarantee-Fees-in-2016.aspx.

5.2 Internally calibrated parameters

The remaining 15 parameters are chosen by minimizing the weighted distance between an equal number of empirical targets and their equivalents in the stationary distribution of the model. The targets fall into three categories. I calculate moments relevant to the mortgage market from the Freddie Mac dataset. I calculate cross-sectional and life-cycle moments of household balance sheets from the 2016 SCF. I take two moments, the mean foreclosure and housing depreciation rates, from aggregate data. As is standard in the literature, the parameters are jointly identified by moments in the data. The following discussion describes how some empirical moments are more or less informative about certain parameters.

Preferences and endowments

The calibration assigns a lower utility weight on nondurable consumption α_j to working relative to retired households (0.709 versus 0.872, respectively). Together, they are informed by the ratio of aggregate housing wealth to income. The subjective discount factor β is 0.920, similar to other values found in the literature, and is pinned down by the observed accumulation of liquid assets relative to income. The ratio of retired to working households' net worth pins down the strength of the bequest motive B, which is equal to 30.825. The flow utility loss from default ξ is 25.882 and is informed by the aggregate foreclosure rate.

Housing

Three parameters govern the distribution of owner-occupied house sizes. The minimum house size $\underline{h}$ is 6.849, the ratio of maximum to minimum house sizes h_{gap} is 1.385, and the spacing parameter for the housing grid h_{skew} is 1.550.[24] Following Kaplan, Mitman and Violante (2020), these parameters are pinned down by the 10th and 90th percentiles of the distribution of home equity relative to net worth. This ensures the model is consistent with observed cross-sectional variation in the reliance of households on home equity for savings.

The large depreciation shock $\bar{\delta}$ is set to 0.24. Negative equity is a necessary but not sufficient condition for default in this class of models. Because the LTV constraint prevents households from beginning their tenure as owners with negative equity and house prices in the model are mean reverting, a large depreciation shock acts as a direct shock to expenditures that may make default optimal for some underwater owners.[25] The probability of a large

[24] Letting N_h be the number of housing grid points and $\bar{h} \equiv h_{gap}\underline{h}$, the value of the ith grid point is $h_i = \left((i-1)\,/\,(N_h-1)\right)^{h_{skew}} \left(\bar{h} - \underline{h}\right) + \underline{h}$. To place the magnitude of $\underline{h}$ in context, its average value in the stationary distribution of the economy is 3.6 times greater than mean household income.

[25] This is the "double trigger" hypothesis of default—see, e.g., Foote, Girardi and Willen (2008).

depreciation shock ζ is chosen so that the average housing depreciation rate is 0.027.[26] This implies that an owner experiences a large depreciation shock once every 19 years on average. Note that, in the stationary distribution of the model, only 3% of homeowners who experience a large depreciation shock choose to default. The housing adjustment cost κ_h is 0.610, and the rent-price ratio R is 0.813. These parameters are informed by the homeownership rate and the fraction of owners with mortgage debt.

Mortgages

The default risk threshold for a high-DTI loan Ψ is 0.050%. The loan adjustment cost κ_m is 0.361, equal to 6% of the average mortgage balance at origination.[27] The loan servicing cost ϕ of 1.29% is pinned down by the mean spread between the mortgage interest rate at origination and the risk-free rate. I set the cost of foreclosure to lenders γ equal to the calibrated value of the housing adjustment cost because, in reality, lenders are responsible for selling properties repossessed after foreclosure.

5.3 Model fit

Table 2 lists internally calibrated parameters, the empirical targets, and their model equivalents. Overall, the model does well at matching the targeted moments, including the aggregate foreclosure and homeownership rates. Crucially, it matches the share of borrowers with DTI ratios above 45%; the share of borrowers with DTI ratios between 43% and 45%; and the mean DTI ratio at origination—key features of the empirical distribution of DTI ratios.

The share of mortgages with a DTI ratio above 45% is highly informative about the parameterization of the default risk threshold Ψ. To place this figure in perspective, around one-quarter of households who obtain new loans have a default probability greater than the threshold value. The observed share of borrowers bunched at the DTI limit is informative about the minimum house size. Intuitively, a larger minimum house size increases loan balances and DTI ratios at origination, but this effect is non-monotonic. If the minimum house size—and, by implication, the required down payment—becomes too large, then some marginal homeowners rent instead. The remaining homeowners have higher incomes and net

[26]See https://apps.bea.gov/scb/account_articles/national/0597niw/tablea.htm.

[27]This is in line with other values from the literature. Berger and Vavra (2015) estimate that the fraction of the value of durable goods lost to adjustment costs is 5.3%. The fixed cost of obtaining a new loan in Boar, Gorea and Midrigan (2022) is 2.3% of mean per-capita income, but their model also features an i.i.d. utility loss from refinancing. Keys, Pope and Pope (2016) document that many homeowners do not refinance when theory predicts they should, so large transaction costs are needed to rationalize observed refinancing behavior.

Table 2. Internally calibrated parameters

	Description	Value	Target	Data	Model
$\alpha_{j<T_R}$	Cons. utility weight when working	0.709	Aggregate housing stock	2.57	2.54
$\alpha_{j\geq T_R}$	Cons. utility weight when retired	0.872	DTI ratio at origination	0.35	0.38
B	Bequest weight	30.825	Ret. to working HHs net worth	1.94	1.93
β	Discount factor	0.920	Aggregate net worth	2.64	2.51
$\bar{\delta}$	High depreciation rate	0.240	Mean LTV ratio	0.37	0.46
h_{gap}	Ratio of max. to min. house size	1.385	90th pctile. home eq. share	1.00	0.98
h_{skew}	Housing grid skewness	1.550	% loans with DTI $\in (43, 45]$	10.24	9.48
$\underline{h}$	Smallest house size	6.849	10th pctile. home eq. share	0.47	0.30
κ_h	Housing transaction cost	0.610	Homeownership rate	0.65	0.63
κ_m	Loan transaction cost	0.361	Share of owners with debt	0.65	0.66
ϕ	Loan servicing cost (%)	1.286	Mean mort. interest rate (%)	2.62	2.60
Ψ	Default risk threshold (%)	0.050	% loans with DTI > 45	8.71	8.33
R	Rent-price ratio	0.813	Aggregate liquid wealth	0.98	1.03
ξ	Utility loss from def.	25.882	Foreclosure rate (%)	0.70	0.80
ζ	High depreciation shock prob. (%)	5.275	Mean depreciation rate (%)	2.27	2.27
γ	Lender foreclosure cost	0.610	Equal to housing adjustment cost κ_h		

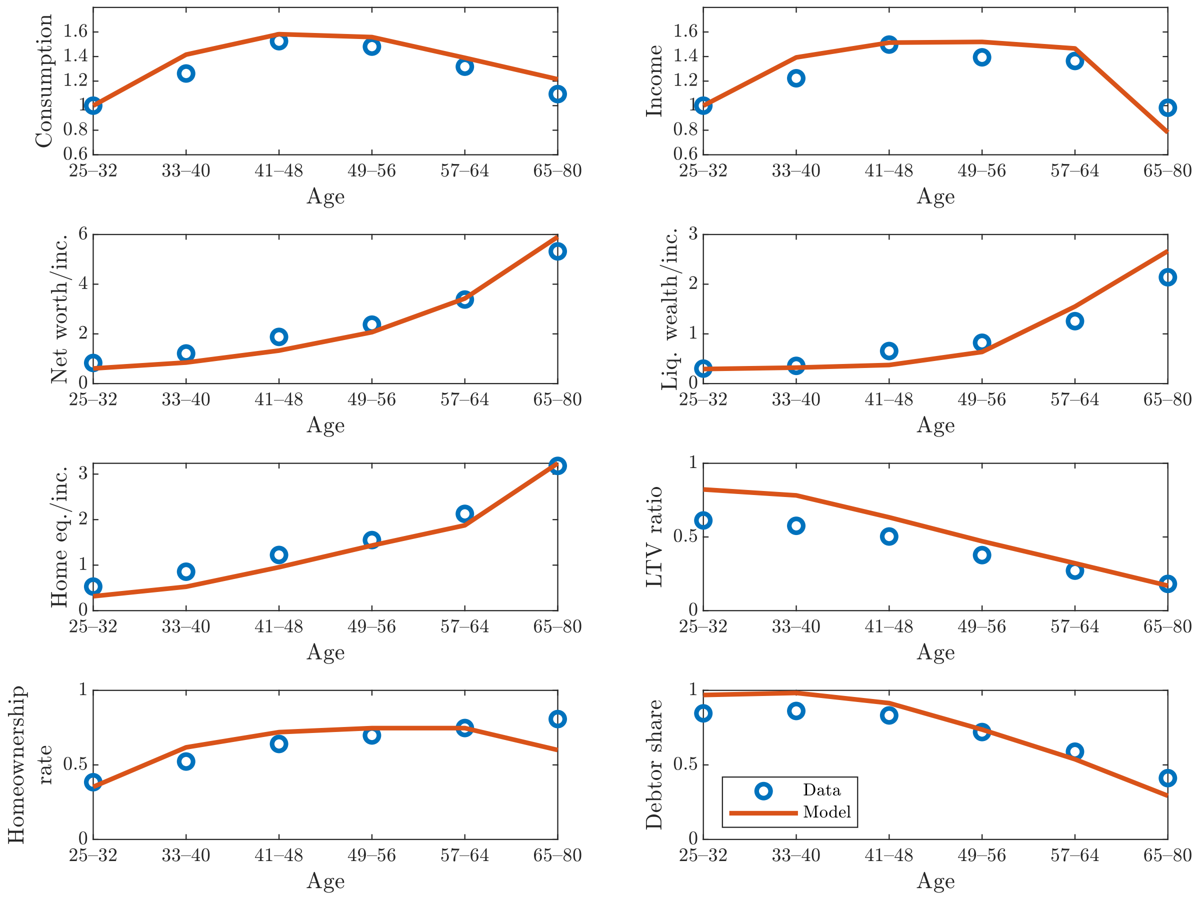
Notes: Aggregate housing stock, net worth, and liquid wealth are scaled by aggregate income. LTV ratio is outstanding debt relative to home value conditional on owning a home in the cross section, not at loan origination. Home equity share is home equity relative to net worth.

worth, so loan balances and DTI ratios at origination decline in equilibrium.

The relatively lower weight on housing services in the utility function of retired households helps the model match the mean DTI ratio at origination. All else equal, retired households with a high preference weight on housing services take out larger loans that are amortized over a relatively short remaining lifetime, implying large payments each period. Retired households also have lower incomes. Reducing retired households' utility weight on housing services prevents the model from generating counterfactually large DTI ratios. This feature of the model could be ameliorated by making loan maturity independent of age, but doing so requires tracking another state variable.

As shown in Figure 5, the model captures important cross-sectional and life-cycle dimensions of household income, consumption, and savings. Parameters governing the income process in the model are directly assigned, so it is unsurprising that the model reproduces life-cycle income profiles. The calibration does not target any moments related to consumption, however. With the exception of the net worth ratio of retired to working households, life-cycle patterns of wealth are also not explicitly targeted in the calibration. The model matches well the increase in and composition of net worth over the life cycle. It is also broadly consistent with homeownership rates and the extensive margin of debt as households age. In the model, homeownership decreases slightly in retirement but remains

FIGURE 5. Household consumption, income, and balance sheets over the life cycle



Notes: Nondurable consumption and income are both normalized relative to households with heads between the ages 25–32. LTV ratio is outstanding mortgage debt scaled by value of the primary residence conditional on homeownership. Debtor share is the fraction of homeowners with positive mortgage debt. Data source: 2016 SCF for household income and net worth, 2017 PSID for nondurable expenditures.

elevated in the data. This stems from a tension in the calibration that arises from retired agents having a lower utility weight on housing services than working agents, which all else equal reduces homeownership at the end of life.

6 Quantitative results of the model

Having validated that the calibrated model matches the empirical distribution of borrowers' debt payment-to-income ratios and the life-cycle profiles of consumption and savings, I now establish that it is consistent with the stylized facts documented in Section 3. Note that, by design, the model delivers an equilibrium in which only households whose default risk is sufficiently low choose high-DTI loans. The quantitative success of the model rests on whether it can yield the observed *difference* in default risk around the DTI limit.

The model replicates quantitatively the discontinuity in default risk at the 45% DTI requirement that is observed in the data. The underwriting technology that gives rise to a soft DTI constraint is necessary for this outcome: without it, the model predicts that default risk is strictly increasing in the DTI ratio at origination. By uncovering the characteristics and counterfactual loan choices of households who are constrained by the DTI limit, the model rationalizes the heterogeneity in their ability to repay.

6.1 Heterogeneity of default risk with respect to DTI ratios

First, I show that the model matches the nonlinear relationship between default risk and DTI ratios at origination in the neighborhood of the current 45% DTI limit. Given that parameters are calibrated in part to match the observed shares of borrowers with DTI ratios above and just below the limit, the selection of relatively creditworthy households into high-DTI loans is an expected outcome of the model. What the calibration leaves open is the creditworthiness of borrowers above the DTI limit *relative* to borrowers below the limit. It also places no explicit restrictions on the characteristics of households found immediately below the 45% cutoff in the stationary distribution of the model.

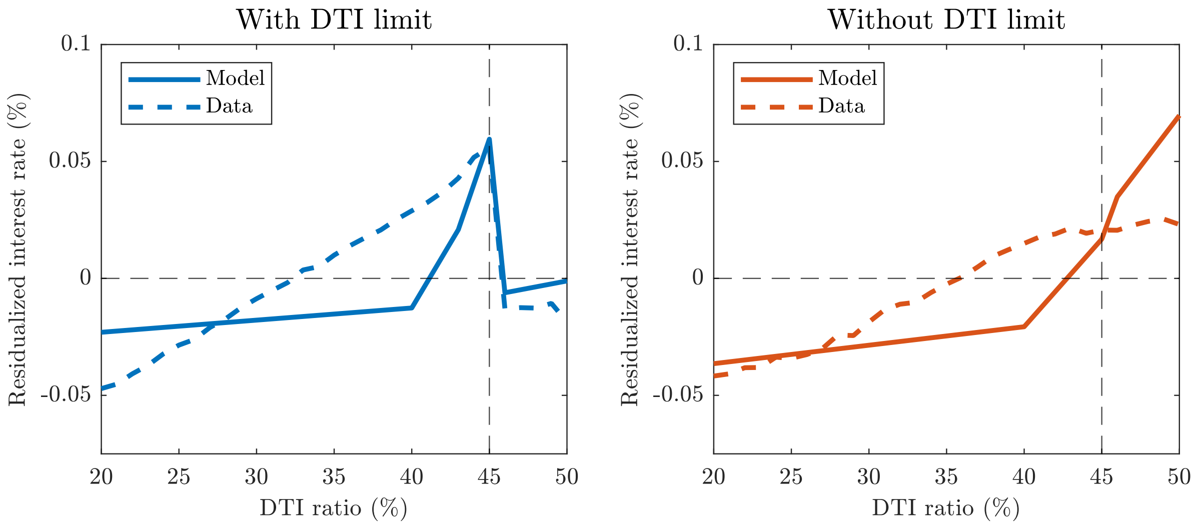
To assess the fit of the model in this regard, I compare how default risk varies with respect to DTI ratios at origination in the model and the data. Because the model lacks a formal notion of credit scores and *ex ante* default probabilities are unobservable in the data, I use mortgage interest rates as a proxy for default risk.[28] As seen in the left-hand panel of Figure 6, the model does well in generating a discontinuity in interest rates at the DTI requirement in line with the empirical estimates from Section 3.2. In the model, the average interest rate on mortgages with DTI ratios between 43% and 45% is 6.5 basis points greater than that on mortgages with DTI ratios above the limit. In the data, this difference is 6.8 basis points. The interest rate on high-DTI loans is 0.6 basis points below average, versus 1.3 basis points in the data.

The underwriting technology is essential for generating this relationship between default risk and DTI ratios in the model. The right-hand panel of Figure 6 plots the equilibrium relationship between interest rates and DTI ratios in a version of the model in which the default risk threshold for high-DTI loans is entirely relaxed and all other parameters are unchanged. This collapses my model to the standard theoretical framework in which households face only a loan-to-value constraint when obtaining a new loan.[29] The standard

[28]See Bosshardt, Kakhbod and Kermani (2023) for evidence that interest rates, net of guarantee fees, on mortgages acquired by the GSEs are predictive of default risk.

[29]I obtain this by setting the default risk threshold Ψ equal to 1 so that it never binds, i.e., no household's loan choice is constrained by the their DTI ratio.

FIGURE 6. Interest rates vs. DTI ratios at origination in the model and data



Notes: "With DTI limit" refers to the baseline calibration of the model. "Without DTI limit" refers to an alternative calibration in which the default risk threshold is fully relaxed but all other parameter values are unchanged. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.

framework cannot generate non-monotonicity in the relationship between borrowers' default risk and DTI ratios. Instead, default risk is strictly increasing in the DTI ratio at origination, qualitatively replicating the correlation that existed when mortgage underwriting standards did not include a DTI requirement. Additionally, this version of the model over-predicts the variance of default risk associated with mortgage originations: relative to the data, too many households with a high propensity to default in their current state enter into new loans. I speculate that, in reality, mortgage lenders informally screenedX applicants in a way that limited the overall riskiness of originations even in the absence of a formal DTI requirement in the pre-policy period.[30] This suggests that the standard framework requires some additional friction that curtails the entry of the least creditworthy households into the mortgage market.

6.2 Characteristics of DTI-constrained households

Next, I identify the characteristics of households who are constrained by the current DTI limit and explain how their optimal choices rationalize the nonlinear relationship between default risk and DTI ratios in the data. The advantage of using a structural model is that I

[30] To use one example, Freddie Mac's selling guidelines have long required additional justification from a lender when originating a mortgage with a DTI ratio above 36%.

TABLE 3. Characteristics and loan choices of DTI-constrained households

	DTI > 45	DTI $\in (43, 45]$	Rent
Income	1.20	1.00	0.98
Liquid wealth	1.57	0.97	1.75
Home equity	2.34	1.25	0.60
Net worth	3.91	2.21	2.35
Owner share	0.99	0.94	0.57
Min. loan payment	0.60	0.25	0.28
Maintenance costs	0.14	0.11	0.23
	Low-DTI loan		
Loan size	3.02	4.04	3.26
Int. rate (%)	2.60	2.64	2.57
Def. prob. (%)	7.48	1.74	7.42
Liq. savings	0.43	0.70	0.04
Consumption	0.38	0.86	−0.09
	High-DTI loan		
Loan size	4.82	4.40	5.97
Int. rate (%)	2.62	2.65	2.74
Def. prob. (%)	0.00	1.71	2.68
Liq. savings	1.79	1.03	1.15
Consumption	0.70	0.89	0.34
% of households	3.10	0.46	0.76

Notes: "DTI > 45" are high-DTI borrowers. "DTI $\in (43, 45]$" are DTI-constrained agents who choose a loan with DTI ratios between 43% and 45%. "Rent" are DTI-constrained agents who rent. Income, balance sheet, and expenditure variables are in units of the numeraire. Owner share is the fraction of existing homeowners. The bottom half of the table displays optimal choices conditional on obtaining a low- or high-DTI loan. "% of households" is the group's share in the total population of the model.

can compute households' counterfactual loan choices, including those would not be observed in any data on mortgage originations. As a result, I define a *DTI-constrained household* as an agent for whom the DTI limit binds because their endogenous probability of default conditional on choosing a high-DTI loan exceeds the threshold value.

To better understand the trade-offs that confront DTI-constrained households, I first study the characteristics of high-DTI borrowers in the model. As seen in the second column of Table 3, they are virtually all homeowners refinancing their loans. A mortgage with a large DTI ratio is feasible for them not only due to their high net worth but also their substantial home equity. For plausibly calibrated housing adjustment costs, such owners would sell their homes before choosing to default. A high-DTI mortgage is optimal for such agents because they have high housing-related outlays relative to current cash on hand and value the additional liquidity from refinancing for precautionary reasons.[31]

In response to being constrained by the DTI limit, households can either choose a smaller mortgage until their DTI ratio is no greater than 45% or opt out of a new loan altogether. The model shows that this choice is informative about the heterogeneity across DTI-constrained households. Borrowers who respond to the DTI limit along the intensive margin of mortgage debt are also largely homeowners who are refinancing an existing loan (see the third column of Table 3). Compared to high-DTI borrowers, they have lower net worth and income but also face a stream of lower required housing outlays. Because their current and future expected liquidity constraints are not as binding, they can optimally substitute into low-DTI mortgages with smaller balances.

By contrast, constrained households who adjust to the DTI limit along the extensive margin of debt differ significantly in their underlying characteristics (see the fourth column of Table 3). Almost half are existing renters, indicating that the high-DTI loans they would have chosen are home purchase loans. Their highly liquid wealth portfolio corroborates this; a large stock of liquid assets is needed to cover the down payment and adjustment costs associated with buying a house. The indivisibility of owner-occupied housing—captured by a minimum house size in the model—is crucial for this result. It induces lumpy adjustment to housing by effectively bounding the face value of a purchase loan from below. A smaller loan is infeasible for agents who need a mortgage for this reason, as shown by the negative consumption it implies. As a result, these households forgo owning and continue to rent. Among all debtors, new homeowners are most likely to default because they have low initial home equity. This leaves them vulnerable to adverse shocks that can make default optimal.

The variation in the loan adjustment decisions of DTI-constrained households accounts for the discontinuity in default risk around the DTI limit depicted in Figure 6. Households constrained along the extensive margin represent would-be high-DTI borrowers with the highest default risk. Their exit from the mortgage market under the current DTI limit lowers the dispersion in default risk among remaining borrowers. Households constrained along the intensive margin reduce their loan size until their DTI ratio no longer violates the limit. The shift of these borrowers within the distribution of DTI ratios explains why equilibrium default risk is highest precisely at the limit and generates the sharp difference in ability to repay on either side of it.

The results underscore the importance of the extensive margin of mortgage debt when assessing the effects of DTI limits on the allocation of credit, with the model predicting

[31] The higher default risk associated with high-DTI borrowers' counterfactual low-DTI loans reflects the expected positive probability that future non-negativity constraints on consumption bind after housing expenses are met.

that around 60% of DTI-constrained households find it optimal to not get a new mortgage altogether. Existing empirical studies corroborate this insight. For instance, DeFusco, Johnson and Mondragon (2020) document that, in response to the introduction of a DTI limit in the jumbo loan market, the observed decrease in high-DTI loans is greater than what existing estimates of the elasticity of mortgage demand to interest rates would imply. They argue that the large quantity response reflects changes in credit supply. The underwriting technology in my model generates a similar outcome whereby mortgages that lenders judge to be "too risky" are simply not originated in equilibrium.

7 Evaluating the Dodd-Frank ability-to-repay rule

Given that the calibrated model accounts for observed heterogeneity in default risk across borrowers conditional on their debt payment-to-income ratios, I use it to evaluate the aggregate and distributional effects of two proposed versions of the Dodd-Frank Act's ability-to-repay (ATR) rule relative to current GSE underwriting standards.[32] The rule exposes a lender who originates a "risky" loan to greater legal liability in the event of borrower default. Mortgages with large DTI ratios are penalized under the original rule, where those with high interest rates are penalized under the revised rule. There has been significant disagreement among policymakers and market participants about the potential effects of the ATR rule on the price and allocation of credit.

Compared to the current policy, aggregate foreclosure and homeownership rise and are quantitatively similar under both implementations of the ATR rule. Despite this, the aggregate welfare gain is higher under the revised price-based version. Intuitively, this is because the stationary distribution features more high-DTI than high-price mortgages. Thus, the original rule generates greater total credit losses from default and a higher equilibrium tax rate under the original rule. The optimal housing tenure choices of households are highly informative regarding the distributional effects of the reforms: welfare gains are especially large for renters constrained along the extensive margin of debt by the current DTI limit who instead choose to own under the ATR rule.

7.1 Description of the Dodd-Frank ability-to-repay rule

The ATR rule requires mortgage lenders to make a "reasonable and good faith determination" at origination of a consumer's ability to repay the loan with the stated goal of reducing mortgage default. To implement the rule, the Consumer Financial Protection Bureau (CFPB) created a category of loans called qualified mortgages (QMs) that are

[32]See Sections 1411–1412 of the Dodd-Frank Act for the text of the ability-to-repay rule.

presumed to comply with it because they lack certain risky features. Loans outside this category are not banned outright, but a lender that originates a non-qualified mortgage faces greater legal liability should the borrower default on it.

In order for the model to capture this trade-off, the payoff that the lender receives from default must be a function of the mortgage's characteristics at the time of origination. Let $q \in \{L, H\}$ denote a qualified and non-qualified mortgage, respectively. When a household obtains a new loan, its characteristics imply a value for q that is fixed for the mortgage's duration. In other words, an age-j household's state vector now includes q.

Under the *original DTI-based rule*, a loan is a QM if the DTI ratio at origination is 43% or less, i.e.,

$$q'_j(\omega) = \begin{cases} L & \text{if } \pi_{min,j}\left(m', r'_{m,j}(\omega)\right) \leq (0.43 - \varsigma_j(\omega))\, y_j(z) \\ H & \text{if } \pi_{min,j}\left(m', r'_{m,j}(\omega)\right) > (0.43 - \varsigma_j(\omega))\, y_j(z)\,. \end{cases}$$

Under the *revised price-based rule*, a loan is a QM if the difference between its annual interest rate and the average prime offer rate is 1.5 percentage points or less.[33] Because the CFPB defines the average prime offer rate as the rate offered to "highly qualified borrowers," I use $(1+g)(1+r+\phi)-1$—i.e., the equilibrium mortgage rate obtained by a household with zero expected probability of default—as its model equivalent.[34] In this case,

$$q'_j(\omega) = \begin{cases} L & \text{if } r'_{m,j}(\omega) - [(1+g)(1+r+\phi)-1] \leq 0.015 \\ H & \text{if } r'_{m,j}(\omega) - [(1+g)(1+r+\phi)-1] > 0.015. \end{cases}$$

Next, because a lender incurs the additional cost of originating a non-qualified mortgage only in states of the world where the borrower defaults, the cost of foreclosure to a lender now depends on q according to

$$\gamma(q) = \begin{cases} \gamma_L & \text{if } q = L \\ \gamma_H & \text{if } q = H. \end{cases}$$

The assumption that $\gamma_H > \gamma_L$ captures the differential legal treatment of qualified and non-qualified mortgages under the ATR rule. By reducing the payoff that a lender receives

[33]See https://www.federalregister.gov/documents/2013/01/30/2013-00736/ability-to-repay-and-qualified-mortgage-standards-under-the-truth-in-lending-act-regulation-z for the original QM definition and https://www.federalregister.gov/documents/2020/12/29/2020-27567/qualified-mortgage-definition-under-the-truth-in-lending-act-regulation-z-general-qm-loan-definition for the revised QM definition.

[34]See https://www.consumerfinance.gov/ask-cfpb/what-is-a-higher-priced-mortgage-loan-en-1797. The model calibration implies a mortgage must have an interest rate no greater than 4.06% at origination in order to meet the revised rule's qualified mortgage definition.

Table 4. Aggregate effects of the ability-to-repay rule

	Current	Original	Revised
Default rate (%)	0.80	0.99	0.99
Homeownership rate	0.63	0.66	0.66
Share of owners with mortgage	0.66	0.69	0.69
LTV ratio at origination	0.70	0.70	0.70
DTI ratio at origination	0.38	0.40	0.40
Mortgage interest rate (%)	2.60	2.63	2.61
Share of mortgages with DTI > 43	0.18	0.18	0.17
Share of high-price mortgages (%)	0.22	0.49	0.44
Aggregate net worth	2.51	2.51	2.51
Aggregate liquid wealth	1.03	1.03	1.03
Aggregate home equity	1.48	1.47	1.47

Notes: "Current," original," and "revised" refer to stationary distribution of the model under the baseline calibration and the two version of the ability-to-repay rule. Loan characteristics are calculated at origination. A high-price mortgage is a non-qualified mortgage under the revised rule. Aggregate net worth, liquid wealth, and home equity are relative to mean income.

from default on a non-QM—see the second line of Equation (10)—the ability-to-repay rule potentially increases the interest rate on this riskier category of loans. All else equal, the ATR rule also increases the credit loss from default on a non-QM—see Equation (13)—which may affect households indirectly by increasing the credit risk guarantees that the government must finance.

To parameterize the foreclosure cost on a qualified mortgage γ_L, I follow the existing calibrations strategy and set it equal to the calibrated value of the housing adjustment cost κ_h in Table 2 because the ATR rule does not change the legal treatment of these mortgages. To parameterize the foreclosure cost on a non-QM loan γ_H, I follow a cost-benefit analysis by the Consumer Financial Protection Bureau (2013) that estimates the monetary cost to a mortgage lender of a legal proceeding brought under the ATR rule. This procedure, described in Appendix B.6, yields a value of 2.434 for γ_H. This is about 4 times larger than γ_L.

In the counterfactuals that follow, I compute and compare the stationary distributions of the model under three policy regimes: the current DTI limit, the original DTI-based ATR rule, and the revised priced-based ATR rule. All welfare calculations abstract from transition dynamics.

7.2 Aggregate effects of the Dodd-Frank ability-to-repay rule

Table 4 presents aggregate outcomes in the steady state of the model under the two policy reforms and the current policy. Relative to the current DTI limit, both versions of

Table 5. Welfare effects of the ability-to-repay rule

	Original	Revised
Welfare change (%)	0.09	0.23
Endogenous tax rate (bps)	31	16
Welfare change (%)	0.31	0.30
Fixed tax rate (bps)	8	8

Notes: Aggregate welfare is reported in consumption-equivalent terms. Under the fixed tax rate, I solve for the stationary distribution of the economy under the two versions of the ability-to-repay rule with the tax rate held at its equilibrium value under the current policy. Under the endogenous tax rate, I do so allowing the tax rate to adjust so that the government's budget constraint for mortgage guarantees in Equation (12) holds.

the ability-to-repay rule result in higher homeownership and default rates. The aggregate default rate increases from 0.8% to 1.0%, driven by a 3-percentage point increase in the homeownership rate. To put this figure into perspective, the U.S. homeownership rate rose 5 percentage points between 1994–2004.[35]

Two mechanisms drive these results. First, compared to the institutional status quo, the ATR rule effectively relaxes credit constraints on households because some currently DTI-constrained households nevertheless exhibit relatively low propensity to default in equilibrium. Under the ATR rule, many such agents use non-qualified mortgages to become homeowners. Their selection into homeownership increases the average default rate among borrowers. Second, because the payoff that lenders expect to receive from default on these mortgages is lower, equilibrium mortgage rates are also higher.

Theoretically, the model's predictions for the size and direction of the ability-to-repay rule's effect on aggregate welfare are ambiguous. Increasing access to homeownership tends to be welfare improving, all else equal. In settings with incomplete markets, housing wealth is a means of self-insurance despite its illiquidity and has additional value of as collateral for mortgage debt. However, because any expansion in homeownership increases the frequency of default among borrowers, total credit losses from default are greater and tax revenue collected from households to fund credit guarantees are higher in equilibrium. The exact magnitude of the aggregate welfare change therefore depends on benefits from greater homeownership relative to costs imposed by more widespread default. Welfare gains from the former are large but concentrated among a small subset of households. Welfare losses from the latter are generally small but affect all households in the economy.

Both versions of the ATR rule improve aggregate welfare but differ in their exact quantitative predictions. As seen in the top half of Table 5, aggregate welfare rises by 0.09%

[35]See https://fred.stlouisfed.org/series/RSAHORUSQ156S.

Table 6. Welfare changes by housing tenure choice under the ability-to-repay rule

	Original Rule		Revised Rule	
	Share of HHs (%)	CEV (%)	Share of HHs (%)	CEV (%)
Own to Own	62.78	−0.00	62.81	0.12
Own to Rent	0.35	−4.72	0.32	−4.14
Rent to Own	0.76	14.87	0.79	15.04
Rent to Rent	36.11	−0.01	36.09	0.14

Notes: "Own" and "rent" refer to a household's housing tenure choice in their current idiosyncratic state. "CEV" is consumption-equivalent welfare change.

in consumption-equivalent terms under the original DTI-based rule, whereas the average welfare gain under the revised price-based rule is more than twice as large at 0.23%. Although both yield identical foreclosure rates, total credit losses from default are higher under the original rule because the incidence of the legal penalty on non-qualified mortgages is more widespread. With the original rule in place, nearly 20% of mortgages are originated with a DTI ratio above 43% and classified as non-QMs. Meanwhile, only 0.4% of newly originated mortgages would be considered non-QMs under the revised price-based rule. Consequently, the original ATR rule implies a higher equilibrium tax rate compared to the revised rule—31 versus 16 basis points, respectively. To demonstrate the importance of accounting for the effect of aggregate credit losses from default through this channel, I compute the aggregate welfare change under both versions of the ATR rule while holding the income tax rate fixed at its equilibrium value under the current DTI limit. I show in the bottom half of Table 5 that the aggregate welfare gain in both cases is now considerably larger, around 0.30% in consumption-equivalent terms. These findings underscore the trade-off policymakers face between reducing mortgage default and promoting access to homeownership.

7.3 Heterogeneous effects of the Dodd-Frank ability-to-repay rule

The optimal housing tenure choices of households are highly informative regarding the heterogeneous effects of the ability-to-repay rule on welfare. Table 6 displays the consumption-equivalent welfare change for households conditional on their housing tenure choices in the baseline model and under the ATR rule. The largest welfare gains accrue to households who switch from renting under the current policy to owning under the ATR rule, whereas the largest welfare losses are experienced by households who make the opposite switch. The vast majority of households' housing tenure choices are unchanged. They are affected by the reform primarily through their continuation values because borrowing constraints are effectively looser in future states of the world. The difference in welfare

Table 7. Loan choices of marginal homeowners under the ability-to-repay rule

	Original Rule		Revised Rule	
	Own to Rent	Rent to Own	Own to Rent	Rent to Own
Income	2.07	1.50	2.12	1.51
Liquid wealth	1.54	1.75	1.64	1.73
Home equity	0.44	0.70	0.38	0.70
Net worth	1.97	2.44	2.03	2.43
Owner share	0.15	0.59	0.12	0.59
	Old loan			
Loan size	8.96	5.03	9.39	5.03
Int. rate (%)	2.86	3.95	2.87	3.95
High-DTI share	0.79	0.89	0.86	0.89
High-price share	0.11	0.62	0.10	0.62
Liquid savings	0.50	0.06	0.53	0.06
Consumption	0.49	0.03	0.52	0.03
	New loan			
Loan size	8.11	7.03	8.37	7.11
Int. rate (%)	3.11	3.02	3.20	2.91
High-DTI share	0.05	0.76	0.03	0.77
High-price share	0.21	0.09	0.26	0.10
Liquid savings	0.15	1.18	0.12	1.14
Consumption	0.35	0.54	0.34	0.53
CEV (%)	−4.72	14.87	−4.14	15.04

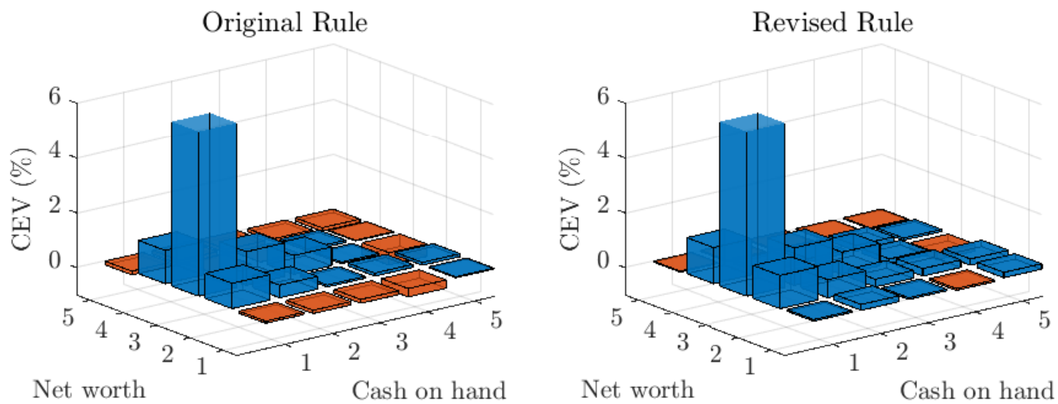
Notes: "Own to rent" ("rent to own") refers to households who choose to own (rent) under the current policy but rent (own) under a given version of the ability-to-repay rule. Income and net worth variables are in terms of the numeraire good. "High-DTI share" is the fraction of households with DTI ratios at origination of 43% or less.

changes between the two versions of the ATR rule reflects the higher equilibrium tax rate needed to balance the government's budget constraint under the original DTI-based implementation.

To illustrate the trade-offs that a given household faces under the ATR rule, Table 7 presents the characteristics and loan choices of marginal homeowners, i.e., the own-to-rent and rent-to-own agents.[36] Around 85% of households who switch from choosing to own to choosing to rent are existing renters. They exhibit the highest default risk among borrowers (see discussion in Section 6.2) and their loan choices under the current policy imply a DTI ratio above 43%. Their equilibrium cost of borrowing rises under the reform because the payoff that lenders would receive from default declines. This endogenously limits how much they can borrow, and they find it preferable to opt out of homeownership altogether. A

[36]The underlying composition and loan choices of marginal homeowners are broadly similar under both versions of the ATR rule.

FIGURE 7. Distribution of welfare changes by net worth and cash on hand



Notes: Net worth and cash on hand quintiles are calculated from the stationary distribution of households under the current policy. "CEV" is consumption-equivalent welfare change. Blue and orange bars indicate a welfare gain and loss, respectively.

large share of households who switch from renting to owning are constrained by the existing DTI limit. Their counterfactual loan choice under the current policy places them near their non-negativity constraint on consumption, which instead makes renting optimal. For these households, the policy reform effectively expands their feasible set of loans because their characteristics imply that they are relatively unlikely to default. Thus, they can increase their desired loan size with limited pass-through of the higher foreclosure cost to their equilibrium cost of borrowing.

To summarize the distributional effects of the ability-to-repay rule, Figure 7 plots consumption-equivalent welfare changes by net worth and cash on hand quintiles. Welfare gains are greatest for households with low cash on hand but medium levels of net worth. They primarily consist of liquidity constrained owners who can more readily extract equity from their homes à la the wealthy hand to mouth in Kaplan and Violante (2014), as well as renters with relatively liquid wealth portfolios who choose a new loan in order to become homeowners. Households with high net worth and cash on hand do not benefit much from the effective relaxation of borrowing constraints because those did not originally bind but do experience a deadweight loss from the higher equilibrium tax rate. Households found near the bottom of the net worth and cash on hand distributions are largely renters, including the marginal homeowners in Table 7.

8 Conclusion

This paper has documented that a statutory limit on debt payment-to-income ratios in place since the Great Recession is a hard constraint for some but not all households. As a result of this policy, a high desired DTI ratio is not necessarily indicative of a high likelihood of default. To rationalize this finding and conduct counterfactual analyses, I introduced a simple loan underwriting technology that captures the discretion of mortgage lenders to relax a DTI constraint for households with sufficiently low endogenous default risk into an otherwise standard structural model of mortgage default. The heterogeneity in default risk across households conditional on their DTI ratios that is generated by the calibrated model cautions against the use of DTI limits as a regulatory tool to lower mortgage default. In a policy counterfactual, I found that a proposed reform to the ability-to-repay rule that penalizes lender who originate high-price loans instead of high-DTI loans leads to higher aggregate welfare.

Looking ahead, there are ways in which the issues raised in this paper could be explored further. Incorporating additional externalities that arise from household leverage and mortgage default would permit a broader examination of why policymakers may want to implement DTI limits in the first place. Shocks to nominal interest rates are an important driver of mortgage refinancing, which is itself relevant channel for the transmission of monetary policy. An environment with an endogenous risk-free rate would be more suitable for studying such questions. Finally, the model developed in this paper could be used to analyze optimal DTI limits.

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A Empirical appendix

A.1 Additional figures

Table 8 contains summary statistics for the sample of Freddie Mac loans used in the empirical analysis in Section 3. Figure 8 plots the mean DTI ratio, high-DTI share, credit score, and interest rate at origination by year. It reveals substantial time-series variation in these loan characteristics. DTI ratios and the share of mortgages with DTI ratios above 45% fell during the Great Recession before recovering somewhat in recent years. Borrowers in the sample are on average more creditworthy now than they were prior to the Great Recession. Mortgage interest rates have trended downward since the mid-2000s.

Figure 9 displays the evolution of the distribution of DTI ratios from 2005 to 2016. There is little visual indication of a binding DTI constraint at any level during the housing boom. Since 2009, the share of mortgages with DTI ratios above 45% has decreased but still account for around 9% of originations purchased or guaranteed by Freddie Mac. The share of mortgages with DTI ratios at or just below the statutory limit has increased over time. As of 2016, around 10% of mortgages have a DTI ratio between 43% and 45%.

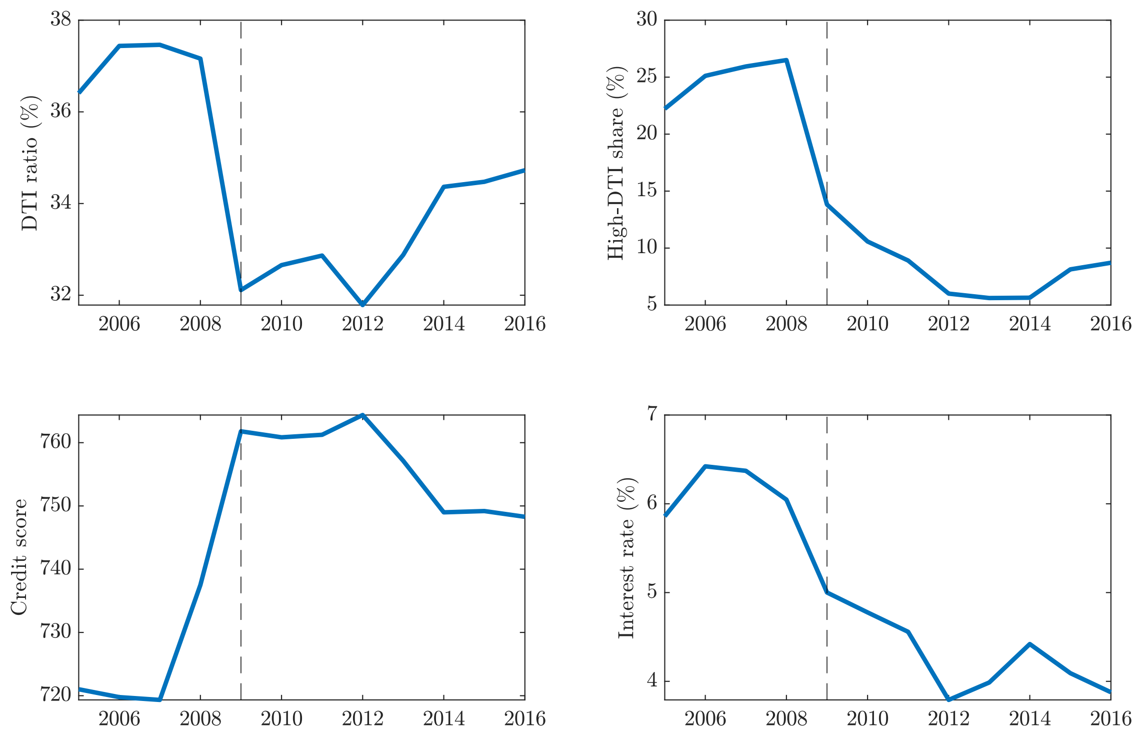
In Figure 10, I plot residualized LTV ratios as a function of DTI ratios separately for loans originated before and after the introduction of the DTI limit in 2009. Prior to the policy, the LTV ratio is initially increasing in the DTI ratio before flattening out. After the policy, loans to the right of the DTI limit have smaller LTV ratios compared to loans just at or below the limit. To the extent that smaller LTV ratios are indicative of lower risk of default, this finding further corroborates the selection of more creditworthy borrowers into high-DTI loans during the post-policy period. It also ensures that the high DTI ratios observed in the post-policy period are not driven by large loan balances.

Table 8. Loan-level summary statistics

Variable	Mean	Std. Dev.	25th pctile.	50th pctile.	75th pctile.
DTI ratio (%)	34.64	10.86	27	35	43
Credit score	744.39	50.54	711	755	785
Interest rate (%)	5.01	1.02	4.13	4.88	5.88
LTV ratio (%)	72.81	16.49	65	78	80
Loan amount (000s)	224.16	115.54	135	200	297

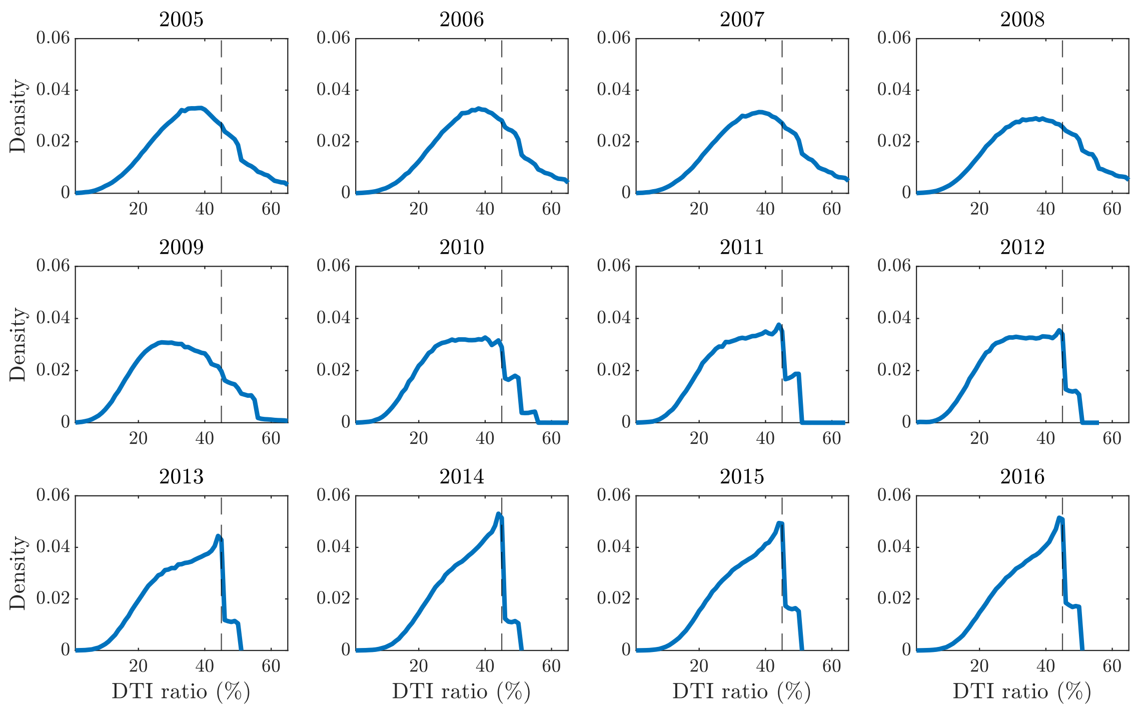
Notes: The DTI ratio, credit score, LTV ratio, and loan amount at origination are reported as whole numbers in the dataset. Source: Freddie Mac Single Family Loan-Level Dataset.

Figure 8. Loan characteristics over time



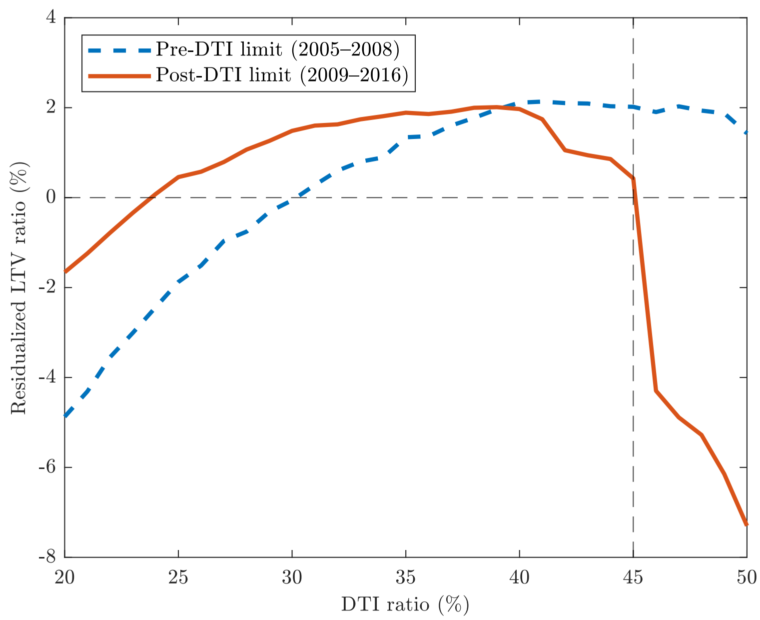
Notes: The vertical dashed black line at 2009 indicates the introduction of the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.

FIGURE 9. Distribution of DTI ratios of loans purchased by Freddie Mac



Notes: Mortgages are grouped into 1-percentage point bins. The dashed vertical line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.

FIGURE 10. LTV ratio vs. DTI ratio at origination



Notes: The LTV ratio is residualized with respect to a vector of year-quarter dummy variables. Averages are computed for each 1-percentage point DTI bin for pre- and post-2009Q1 observations separately. The vertical dashed black line indicates the 45% DTI limit. Source: Freddie Mac Single Family Loan-Level Dataset.

A.2 Difference-in-differences regression

To control for other observable variables that may affect the relationship between DTI ratios and borrower default risk in Section 3.2, I use a difference-in-differences specification common to this literature—e.g., DeFusco, Johnson and Mondragon (2020)—to estimate the change in the characteristics of a high-DTI loan, relative to a low-DTI loan, after the introduction of the 45% DTI limit. The regression equation is

$$y_{it} = \alpha + \beta_1 HighDTI_i + \beta_2 \left(HighDTI_i \times Policy_t\right) + \gamma_t + X_i'\delta + \varepsilon_{it}, \tag{14}$$

where y_{it} is a characteristic of loan i originated in year-quarter t, α is a constant, γ_t is a vector of year-quarter dummy variables, X_i is a vector of loan-specific characteristics, and ε_{it} is an error term clustered at the state level.[37] $HighDTI_i$ equals 1 if the DTI ratio of loan i exceeds 45% and 0 otherwise. $Policy_t$ equals 1 if the year-quarter of origination is 2009Q1 or later and 0 otherwise. The coefficient of interest is β_2, which represents the differential change in the dependent variable for high-DTI loans relative to low-DTI loans after the introduction of the 45% DTI limit. When estimating the regression, I restrict my sample to loans with a DTI ratio between 40% and 50%.[38]

I estimate the regression using the credit score and interest rate as dependent variables. Table 9 displays estimated values for β_1 and β_2 from Equation (14). The correlations documented in Figure 2 survive after controlling for this richer set of covariates and are all statistically significant at the 1% level. Relative to borrowers with low-DTI loans, the credit score of borrowers with high-DTI loans increases by 10.7 after the DTI limit is introduced. The interest rate they receive on their loans correspondingly declines by 5.5 basis points.

Additionally, I estimate the difference-in-differences specification in Equation (14) using the LTV ratio as the dependent variable. The coefficient on the interaction term is negative and statistically significant at the 1% level. Table 10 contains estimated values for β_1 and β_2 and shows that, relative to low-DTI loans, the LTV ratio of high-DTI loans declined by 5.5 percentage points after the DTI limit came into effect.

[37] X_i includes dummy variables for the state in which the property is located, loan purpose (i.e., purchase or refinance), type of property, number of units on the property, and whether the borrower is a first-time homebuyer. Including $Policy_t$ as a separate regressor is unnecessary because it is absorbed by the year-quarter dummy variables.

[38] The results are robust to estimating the regression for the full sample of loans.

TABLE 9. Effect of the 45% DTI limit on default risk and interest rates at origination

	(1) Credit score	(2) Interest rate (%)
DTI > 45%	−1.270***	0.002**
	(0.000)	(0.023)
DTI > 45% × Policy	10.705***	−0.055***
	(0.000)	(0.000)
N	3,061,386	3,061,386
R^2	0.139	0.909

p-level in parentheses.
Robust standard errors are clustered at the state level.
* p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The two rows contain estimates for the coefficients β_1 and β_2 from the specification in Equation (14). Source: Freddie Mac Single Family Loan-Level Dataset.

TABLE 10. Effect of the 45% DTI limit on LTV ratios at origination

	(1) LTV ratio (%)
DTI > 45%	−0.073
	(0.105)
DTI > 45% × Policy	−5.363***
	(0.000)
N	3,061,386
R^2	0.238

p-level in parentheses.
Robust standard errors are clustered at the state level.
* p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The two rows show estimates for the coefficients β_1 and β_2 from the specification in Equation (14). Source: Freddie Mac Single Family Loan-Level Dataset.

To verify that the results are not driven by the DTI ratio on a loan simply being large but rather a change exactly at the statutory limit of 45%, I follow DeFusco, Johnson and Mondragon (2020) in estimating a flexible difference-in-differences specification that allows the effect of the policy to vary with the DTI ratio. The regression equation is

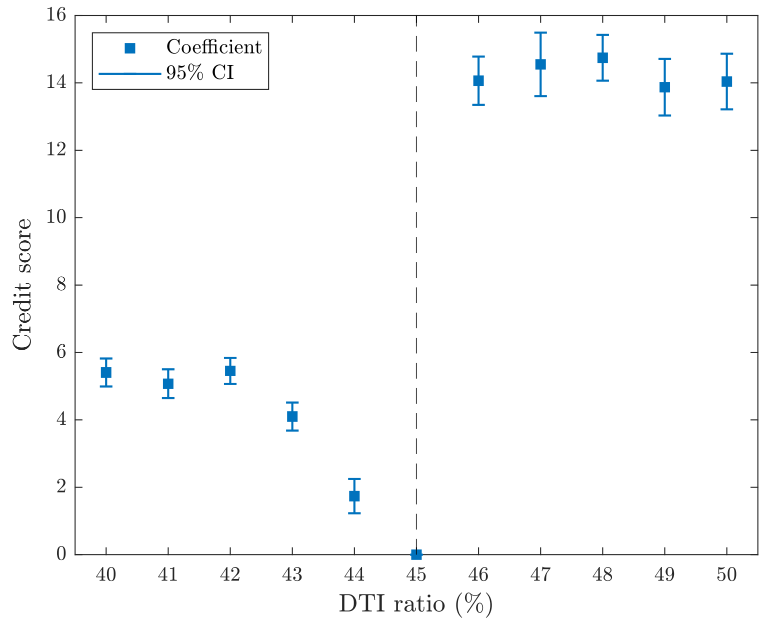
$$y_{it} = \alpha + \sum_{k=40}^{50} \left[\beta_1^k \mathbb{1}_{DTI_i=k} + \beta_2^k \left(\mathbb{1}_{DTI_i=k} \times Policy_t\right)\right] + \gamma_t + X_i'\delta + \varepsilon_{it}, \tag{15}$$

where $\mathbb{1}_{DTI_i=k}$ is an indicator variable that takes a value of 1 if the DTI ratio (in percent) of loan i at origination is equal to $k \in \{41, 42, \ldots, 50\}$ and all other terms are as previously defined in Equation (14). I make $k = 45$ the omitted category such that β_2^k estimates the differential change in y_{it} for loans originated with a DTI ratio equal to k relative to loans with a DTI ratio of 45% after the policy is introduced. I estimate Equation (15) for a sub-sample of loans with DTI ratios between 40% and 50%, and robust standard errors are clustered at the state level.

Figure 11 plots point estimates for $\left\{\beta_2^k\right\}_{k=40}^{50}$ and their respective 95% confidence intervals when Equation (15) is estimated with the credit score as the dependent variable. Relative to borrowers with DTI ratios equal to 45%, the credit score of borrowers who receive high-DTI mortgages increases by 14 after the DTI limit is introduced. Figure 12 plots point estimates for coefficients on the interaction terms and their respective 95% confidence intervals using the interest rate as the dependent variable. Relative to borrowers with DTI ratios equal to 45%, borrowers with high-DTI mortgages receive interest rates that are on average 7 basis points higher after the DTI limit is introduced.

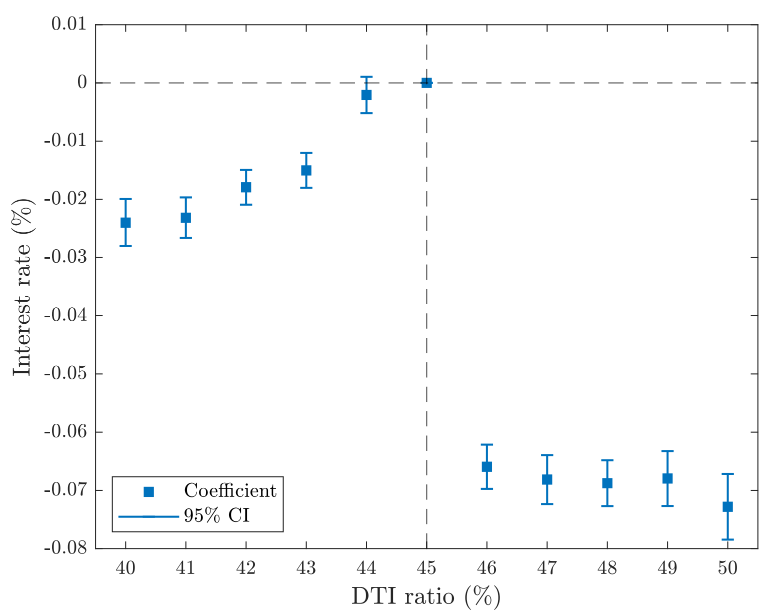
Note that the differential changes in default risk implied by estimates from the flexible difference-in-differences specification are larger than those implied by the baseline specification. This is because the control group in the baseline specification contains all loans with DTI ratios between 40% and 45%. The control group for the flexible difference-in-differences regressions consists only of mortgages with DTI ratios exactly equal to 45% and, as seen in Figure 2, these exhibit the highest likelihood of default in the sample.

FIGURE 11. Effect of the 45% DTI limit on credit scores at origination



Notes: Blue squares correspond to estimates of β_2^k for $k \in \{40, 41, \ldots, 50\}$ from the specification in Equation (15), where $k = 45$ is the omitted category. Blue bars denote the 95% confidence interval around the point estimates. Source: Freddie Mac Single Family Loan-Level Dataset.

FIGURE 12. Effect of the 45% DTI limit on interest rates at origination



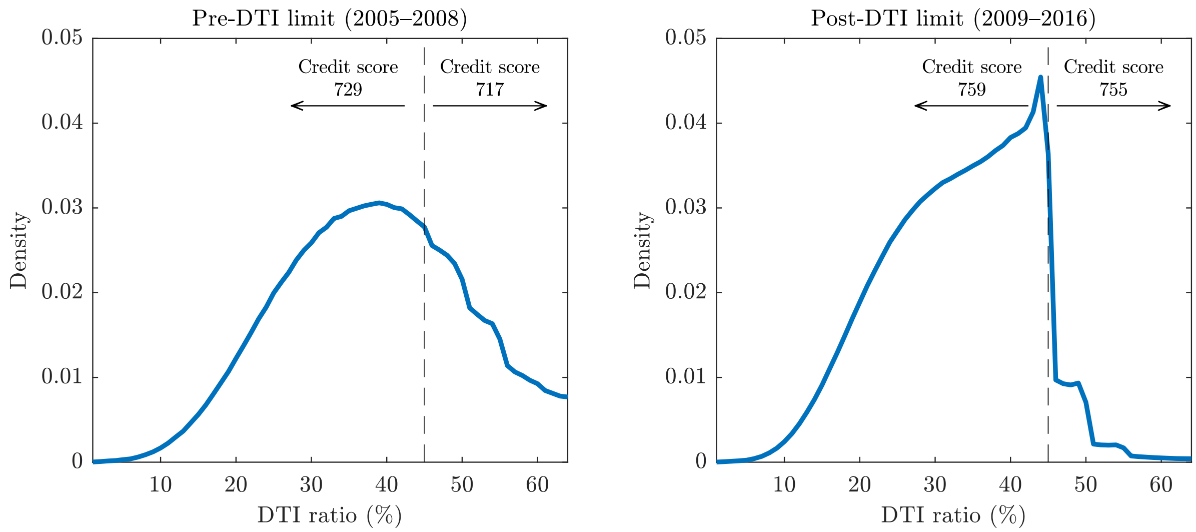
Notes: Blue squares correspond to estimates of β_2^k for $k \in \{40, 41, \ldots, 50\}$ from the specification in Equation (15), where $k = 45$ is the omitted category. Blue bars denote the 95% confidence interval around the point estimates. Source: Freddie Mac Single Family Loan-Level Dataset.

A.3 Robustness: effect of the DTI limit on Fannie Mae loans

The baseline empirical analysis focuses on mortgage originations purchased or guaranteed by Freddie Mac because this is the segment of the mortgage market most directly affected by the 45% DTI limit. There may be concerns about the extent to which the stylized facts in Section 3 can be generalized more broadly to the mortgage market. For instance, during the sample period, the underwriting standards of Fannie Mae featured a DTI limit of 50%. It is possible that mortgage lenders could have avoided Freddie Mac's more strict DTI requirement by selling their originations to Fannie Mae instead. To address this concern, I redo my empirical analysis using the Fannie Mae Single-Family Loan Level Dataset. To ensure comparability with the sample of loans purchased by Freddie Mac, I only include loans purchased or guaranteed by Fannie Mae with 30-year terms that are collateralized by owner-occupied housing and have non-missing data on DTI ratios, FICO scores, interest rates, and LTV ratios. This leaves me with around 23 million observations.

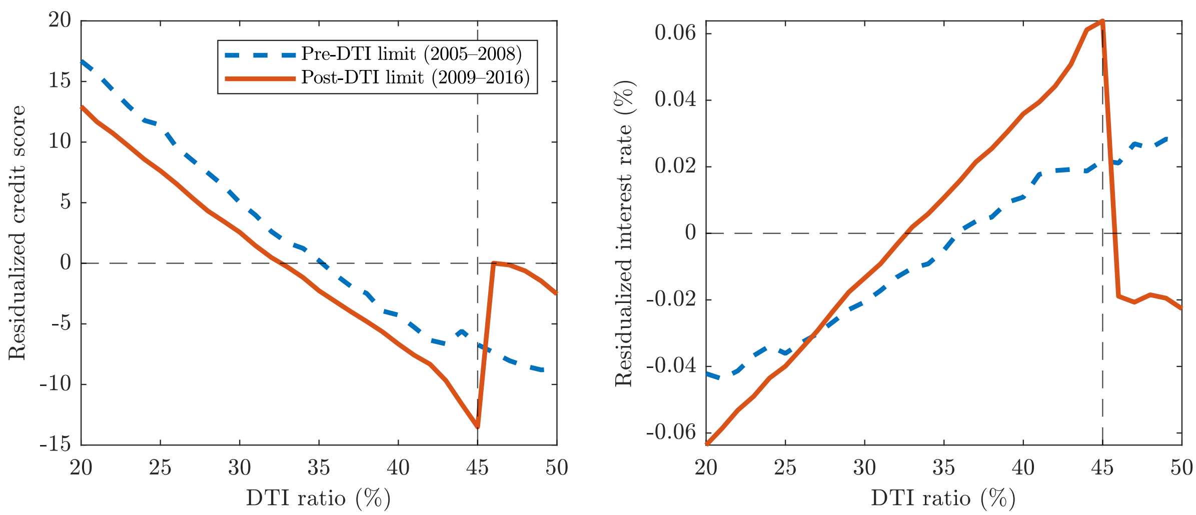
In Figure 13, I verify that both GSEs have operated under a *de facto* DTI limit of 45% during the sample period despite differences in their DTI requirements on paper. After Freddie Mac introduced their 45% DTI limit in 2009, the distribution of DTI ratios among Fannie Mae mortgages also features significant bunching at 45%. Similarly, the difference in average credit scores on either side of the 45% DTI cutoff is much smaller in the post-policy period compared to the pre-policy period. In Figure 14, I verify that, among Fannie Mae loans, those with DTI ratios above the 45% limit are associated with higher credit scores and lower interest rates compared to those just at or below the limit. Thus, the patterns of borrower selection documented among mortgages bought or guaranteed by Freddie Mac hold more broadly for the conforming loan market. I speculate that, with the collapse of private-label securitization and the reluctance of lenders to retain mortgages on their balance sheets after the Great Recession, primary mortgage lenders have an incentive to maximize their ability to sell their originations to either of the GSEs. This may explain why the 45% DTI cutoff is relevant even for Fannie Mae loans.

Figure 13. DTI ratios and credit scores before and after the 45% DTI limit: Fannie Mae loans



Notes: Mortgages are grouped into 1-percentage point bins. The dashed black vertical line indicates the 45% DTI limit. The mean credit score that appears to the left (right) of the dashed black line is calculated from mortgages originated with a DTI ratio less (greater) than 45%. Source: Fannie Mae Single Family Loan-Level Dataset.

Figure 14. Credit scores and interest rates vs. DTI ratios at origination



Notes: The credit score and interest rate are each residualized with respect to a vector of year-quarter dummy variables. Averages are computed for each 1-percentage point DTI bin for pre- and post-2009Q1 observations separately. The vertical dashed black line indicates the 45% DTI limit. Source: Fannie Mae Single Family Loan-Level Dataset.

A.4 Robustness: mortgages with varying maturity

The baseline empirical analysis focuses on mortgages with 30-year terms because they represent the most common residential mortgage contract used in the United States. However, the debt payment-to-income ratio on a mortgage in part depends on its maturity. All else equal, a longer loan term mechanically decreases the size of the debt payment and the DTI ratio. To the extent that the baseline analysis excludes loans with shorter maturities and higher DTI ratios, this may lead the empirical findings to omit the types of households who may be most affected by the introduction of the 45% DTI limit.

Table 11 displays summary statistics for the sample of loans chosen according to the criteria described in Section 3.1 but without the 30-year maturity restriction. The mean term of loans in this broader sample is 26.3 years, and mortgages with 30-year terms account for 74% of them. Of the remaining 26% of loans, the vast majority have shorter terms. Mortgages with terms less than 30 years in fact have somewhat lower DTI ratios, not higher ones: between 2005 and 2016, 9% of these shorter-maturity mortgages are high-DTI loans compared to 15% of 30-year mortgages. Mortgages with shorter maturities are also associated with higher credit scores, smaller balances, and lower interest rates. It is likely that a borrower composition effect can account for the observed positive relationship between loan maturity and DTI ratios in the dataset. Altogether, the summary statistics suggest that the types of households who choose mortgages with shorter terms in equilibrium are unlikely those for whom the 45% DTI limit is likely to bind.

To check that my empirical findings are robust to using a sample of mortgages of varying maturities, I estimate the difference-in-differences specification in Equation (14) without restricting the sample to loans with 30-year terms. As before, I estimate this regression for loans in a symmetric 10-percentage point window around the 45% DTI cutoff and use the credit score and interest rate as the dependent variable. Estimated values for β_1 and β_2 are in Table 12, where the coefficient of interest β_2 captures the differential change in the dependent variable for high-DTI loans relative to low-DTI loans after the introduction of the 45% DTI limit in 2009Q1. The results documented in the main body of the text are largely unchanged when estimated for this sample of loans with varying maturity. Compared to low-DTI loans, the credit score and interest rate on high-DTI loans increases by 9.9 and decreases by 6.7 basis points, respectively. I also verify that the LTV ratio on high-DTI loans decreases by more relative to low-DTI loans following the policy change for mortgages of differing maturities.

Table 11. Loan-level summary statistics for varying loan maturities

Variable	Term < 30	Term = 30	Term > 30	All
DTI ratio (%)	30.62	34.64	42.37	33.57
Credit score	756.27	744.39	708.78	747.53
Interest rate (%)	4.15	5.01	6.55	4.78
LTV ratio (%)	62.13	72.81	74.35	69.97
Loan amount (000s)	179.12	224.16	224.83	212.18
High-DTI share	0.09	0.15	0.40	0.13
Refinance share	0.87	0.56	0.66	0.64
First-time homebuyer share	0.04	0.16	0.13	0.12
N	3,783,071	10,440,989	5,242	14,229,302

Notes: The DTI ratio, credit score, LTV ratio, and loan amount at origination are reported as whole numbers in the dataset. Source: Freddie Mac Single Family Loan-Level Dataset.

Table 12. Effect of the 45% DTI limit on loan originations: all maturities

	(1) Credit score	(2) Interest rate (%)	(3) LTV ratio (%)
DTI > 45%	−1.258***	0.002*	−0.081*
	(0.000)	(0.076)	(0.064)
DTI > 45% × Policy	9.889***	−0.067***	−4.865***
	(0.000)	(0.000)	(0.000)
N	3,849,413	3,849,413	3,849,413
R^2	0.138	0.875	0.227

p-level in parentheses.
Robust standard errors are clustered at the state level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: The two rows contain estimates for the coefficients β_1 and β_2 from the specification in Equation (14) for a broader sample of loans with varying maturity. Source: Freddie Mac Single Family Loan-Level Dataset.

B Model appendix

B.1 Equilibrium definition

To establish notation, I define the state space, holding age j fixed, as $W \equiv A \times H \times D \times P \times M \times R_M \times Z$. Define the σ-algebra Σ_W as $B_A \otimes B_H \otimes B_M \otimes B_{R_M} \otimes P(D) \otimes P(P) \otimes P(Z)$. B_A, B_H, B_M, and B_{R_M} are the Borel σ-algebras on A, H, M, and R_M, respectively, and $P(D)$, $P(P)$, and $P(Z)$ are the power sets of D, P, and Z, respectively. Define $\Omega \equiv \mathcal{A} \times \mathcal{H} \times \mathcal{D} \times \mathcal{P} \times \mathcal{M} \times \mathcal{R}_{\mathcal{M}} \times \mathcal{Z}$ as the typical subset of Σ_W.

For a given parameterization of the model and a measure of age-1 households $\mu_1(\omega)$, a stationary recursive equilibrium consists of

1. household value functions $\left\{V_j^R(\omega), V_j^M(\omega), V_j^P(\omega), V_j^D(\omega)\right\}$ and policy functions $\left\{c_j(\omega), s_j(\omega), a'_j(\omega), h'_j(\omega), m'_j(\omega)\right\}$;
2. a mortgage interest rate schedule $r'_{m,j}(\omega)$;
3. a government tax policy τ; and
4. a stationary measure $\Lambda_j^*(\Omega)$;

such that,

1. given $r'_{m,j}(\omega)$ and τ, household value and policy functions solve the optimization problems in Equations (3), (4), (5), (6), (8), and (9);
2. given household value and policy functions and a government tax policy, $r'_{m,j}(\omega)$ is such that financial intermediaries' zero-expected profit condition in Equation (11) is satisfied on a loan-by-loan basis;
3. given household value functions and mortgage market clearing, τ is such that the government budget constraint in Equation (12) holds; and
4. the invariant probability measure satisfies

$$\Lambda_{j+1}^*(\Omega) = \int_{\Omega} Q_j(\omega, \Omega)\left[\Lambda_j^*(d\omega) + \mu_1(d\omega)\right] \tag{16}$$

   for all $\Omega \in \Sigma_W$, where $Q_j(\omega, \Omega)$ is the conditional probability that an age-j household in state ω transitions to the set Ω at age $j+1$ and defined as

$$Q_j(\omega, \Omega) \equiv \mathbb{1}_{a'_j(\omega)\in\mathcal{A},\, h'_j(\omega)\in\mathcal{H},\, m'_j(\omega)\in\mathcal{M},\, r'_{m,j}(\omega)\in\mathcal{R}_{\mathcal{M}}} \sum_{\delta'\in\mathcal{D}} \sum_{p'\in\mathcal{P}} \sum_{z'\in\mathcal{Z}} \pi(\delta')\, \pi(p'|p)\, \pi(z'|z)\,. \tag{17}$$

B.2 Parameterizing the income process

To parameterize the deterministic age-dependent component of income, I follow Kaplan and Violante (2014) in regressing a quartic polynomial in age on log annual household income of households whose heads are between ages 25–65 from the 1999–2017 waves of the Panel Study of Income Dynamics (PSID). Using the estimated regression coefficients, I compute fitted values for log household income. These fitted values are the sequence $\{\chi_j\}_{j=1}^{T_R-1}$ in the model, normalized so that log income of an age-1 household is 0.

To parameterize the function for pension income $\Phi\left(y_{T_R-1}(z)\right)$, I follow the procedure described by Guvenen and Smith (2014). For a given $(\rho_z, \sigma_\varepsilon)$ pair, I simulate earnings for a panel of 100,000 working-age households and regress their average labor earnings on earnings at age $T_R - 1$. I use the estimated regression coefficients to predict average lifetime earnings $\log \hat{y}$ for each possible realization of income at age $T_R - 1$, i.e., $\log y_{T_R-1}(z) = \chi_{T_R-1} + z$. Let $\log \bar{y}$ be the economy-wide average annual labor earnings and define $\log \tilde{y} \equiv \log \hat{y} / \log \bar{y}$. The function for pension income estimated by Guvenen and Smith (2014) is

$$\Phi\left(y_{T_R-1}(z)\right) = \begin{cases} \log \bar{y}\left[0.9 \log \tilde{y}\right] & \text{if } \log \tilde{y} \leq 0.3 \\ \log \bar{y}\left[0.27 + 0.32\left(\log \tilde{y} - 0.3\right)\right] & \text{if } 0.3 < \log \tilde{y} \leq 2 \\ \log \bar{y}\left[0.81 + 0.15\left(\log \tilde{y} - 2\right)\right] & \text{if } 2 < \log \tilde{y} \leq 4.1 \\ 1.13 \log \bar{y} & \text{if } \log \tilde{y} > 4.1. \end{cases}$$

Figure 15 plots the median life-cycle income profile that results from these procedures.

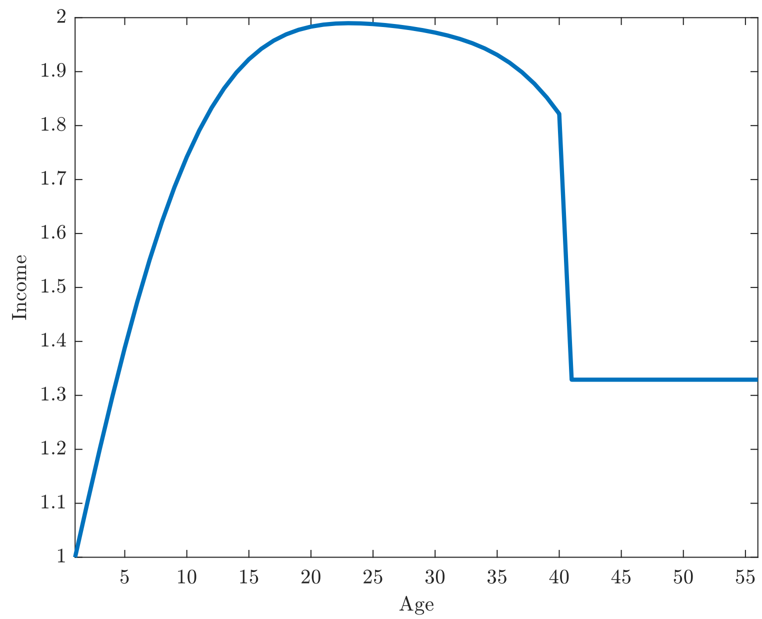
B.3 Estimating the DTI offset

According to the Freddie Mac Single-Family Seller/Servicer Guide, liabilities that must be considered when computing a borrower's debt payment-to-income ratio are their monthly housing expense, payments on all installment debts (e.g., student loans), payments on revolving accounts (e.g., credit cards), child support, and alimony. Monthly housing expense consists of principal and interest payments on the mortgage; property hazard insurance premiums; real estate taxes; homeowners association (HOA) dues; and other expenditures where applicable.[39]

I directly calibrate the DTI offset using the 2016 Survey of Consumer Finances (SCF) and the 2017 American Housing Survey (AHS). The offset is the sum of (1) non-first mortgage debt service payments and (2) other housing expenses, both express relative to income. I use the 2016 SCF to parameterize the first component of the DTI offset. A household's non-

[39]See https://guide.freddiemac.com/app/guide/content/a_id/1000663.

Figure 15. Life-cycle income profile in the model



Notes: This figure plots median age-specific income $\exp(\chi_j)$ and pension income $\Phi(y_{T_R-1}(z))$ for ages 1–40 and 41–56, respectively, in the model.

mortgage DTI ratio is the sum of their non-mortgage revolving and non-revolving ratios; DTI ratios on second and third mortgages; and the ratio of alimony payments to income. To control for the effect that observable household characteristics may have on the non-mortgage DTI ratio, I estimate the regression

$$\varsigma_i = \alpha + \sum_{j=1}^{4} \beta_j age_i^j + \gamma\left(\frac{w_i}{y_i}\right) + \delta\left(\frac{a_i}{y_i}\right) + \zeta \mathbb{1}_{h_i>0} + X_i'\eta + \varepsilon_i, \tag{18}$$

where ς_i is the non-mortgage DTI ratio of household i, α is a constant term, $\left\{age_i^j\right\}_{j=1}^4$ is a quartic polynomial in age, w_i/y_i is the net worth to income ratio, a_i/y_i is the liquid wealth to income ratio, $\mathbb{1}_{h_i>0}$ is an indicator variable for homeownership, X_i is a vector of household demographic characteristics, and ε_i is an error term.[40] I include these three balance sheet variables in the regression because they have direct model equivalents. Estimated regression coefficients are presented in Table 13.

[40] X_i includes indicator variables for the head of household's race, education, sex, marital status, and labor force participation, as well as the number of children in the household.

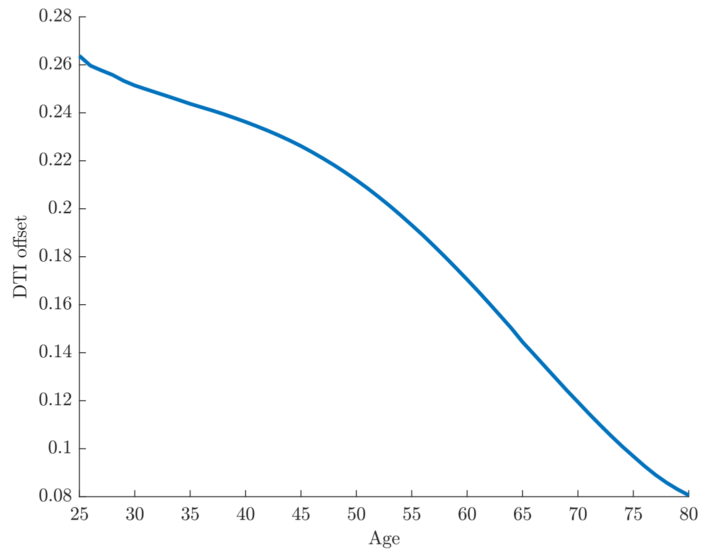
Table 13. Estimated coefficients from the DTI offset regression equation

	(1) Non-mortgage DTI ratio
age	-0.024*
	(0.060)
age^2	0.001**
	(0.043)
age^3	-0.000**
	(0.025)
age^4	0.000**
	(0.014)
w/y	0.000***
	(0.008)
a/y	-0.001***
	(0.000)
$\mathbb{1}_{h>0}$	-0.011***
	(0.000)
Constant	0.410***
	(0.005)
N	16,419
R^2	0.035

p-level in parentheses.
* p < 0.10, ** p < 0.05, *** p < 0.01.

Notes: The rows report estimated values of $\{\beta_j\}_{j=1}^4$, γ, δ, ζ, and α from the DTI offset regression in Equation (18). Source: 2016 SCF.

FIGURE 16. Mean DTI offset by age in the model



Notes: Holding age j constant, the mean DTI offset is calculated by integrating $\varsigma_j(\omega)$ over the stationary distribution of households who obtain a new loan in in their current state.

To parameterize the second component of the DTI offset, I use 2017 AHS. Housing expenses relative to income are the sum of monthly property taxes (PROTAXAMT), home insurance premiums (INSURAMT), HOA fees (HOAAMT), and lot rent (LOTAMT), all scaled by monthly household income (HINCP). For simplicity, I assume that all agents in the model have the same housing expense ratio and set it equal to the average in the data, which is 8%.

When solving the problem of a household who chooses to own and obtain a new mortgage loan in Equation (6), I use the estimated values of α, $\{\beta_j\}_{j=1}^{4}$, γ, δ, and ζ in Table 13 to predict the non-mortgage DTI ratio given the household's age and state. I add to that the housing expense ratio calculated from the AHS. In Figure 16, I show that this procedure yields a DTI offset that, in the stationary distribution of borrowers in the model, is monotonically decreasing in age. On average, the offset across households obtaining new loans is 20.1%.

B.4 Calculating calibration targets from the Survey of Consumer Finances

In order to reflect the most recent financial conditions of U.S. households, I use the 2016 SCF summary extract file to compute cross-sectional and life-cycle moments of household balance sheets targeted in the calibration. Net worth is the sum of liquid assets and home equity. Following Kaplan and Violante (2014), liquid assets are the sum of assets held in checking accounts, savings accounts, call accounts, directly held mutual accounts, directly held bonds, and directly held stocks. Home equity is the difference between the value of primary residential real estate and debt outstanding on the first mortgage secured by the primary residence.[41] I define household income as the sum of wage and salary income; income from unemployment insurance and benefits; and Social Security and pension income. I limit my sample to households whose heads are between ages 25–80, have strictly positive household income, and below the top 1% of the net worth distribution. All moments are calculated using the SCF sample weights.

I use the bottom 99% of the net worth distribution to calculate calibration targets because the SCF over-samples households who are likely to be relatively wealthy in order to increase representation of the upper tail of the wealth distribution and to make possible analyses of less widely held asset classes. These families correspond to the "list sample" because they are selected using specially edited individual tax returns provided by the Internal Revenue Service. In the 2004 SCF, for example, the list sample accounts for only 15% of observations in the bottom 95% of the wealth distribution but 88% of observations in the top 5% percent.[42] In general, households in the right tail of the wealth distribution are not especially dependent on home equity for savings and unlikely to be affected by the policies studied in this paper.

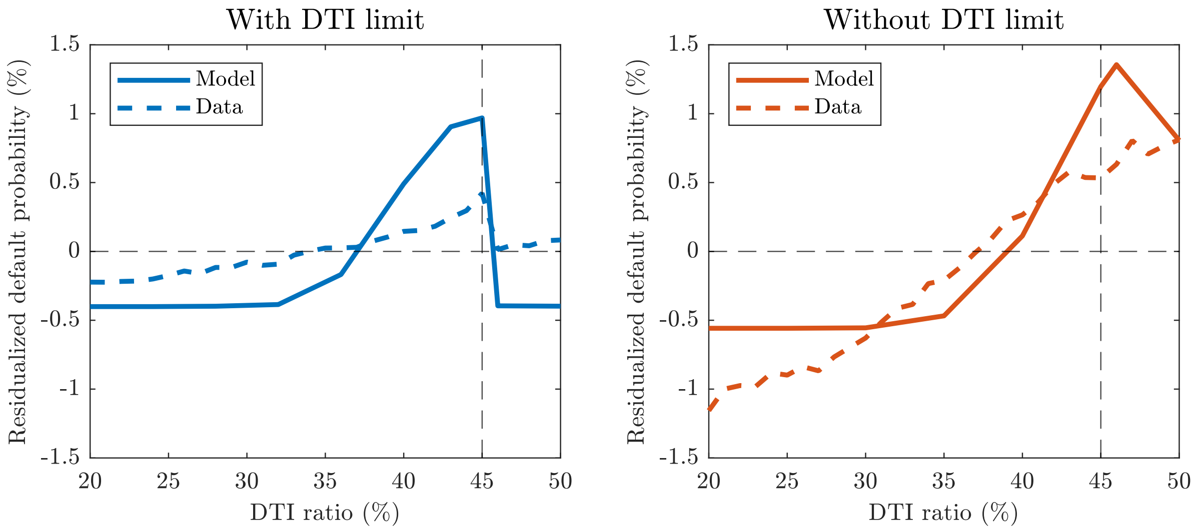
B.5 Additional results on borrower selection into high-DTI mortgages

Figure 17 compares the *ex ante* default probability of borrowers as a function of their DTI ratio in the model to its nearest empirical counterpart, the *ex post* probability that a borrower is delinquent on their mortgage one year after payments begin. I use this comparison because the model does not contain a formal notion of a credit score. A credit score is a backward-looking proxy of a borrower's ability to repay, whereas the default probability in the structural model is a forward-looking measure. To compute the *ex post* delinquency probability, I merge the monthly performance files of the Freddie Mac Single Family Loan-Level Dataset to the quarterly origination files. From the monthly performance files, I construct an indicator variable that equals 1 if a borrower is at least 30 days delinquent on their loan one year after their first payment is due. In reality, borrowers are often delinquent on their payments for some time before a formal foreclosure process is initiated. In the model, delinquency and default are identical decisions: a debtor who does not make a mortgage payment necessarily loses their house as well. Because the definitions of default in the model and the data differ, one should not expect the quantitative magnitudes to line up exactly. Nevertheless, the model generates predictions that are qualitatively similar to

[41] Because relatively few households in the SCF report having more than one loan secured by their primary residence or owning a second home, the inclusion of second mortgages (e.g., home equity loans or home equity lines of credit) is quantitatively unimportant for my results.

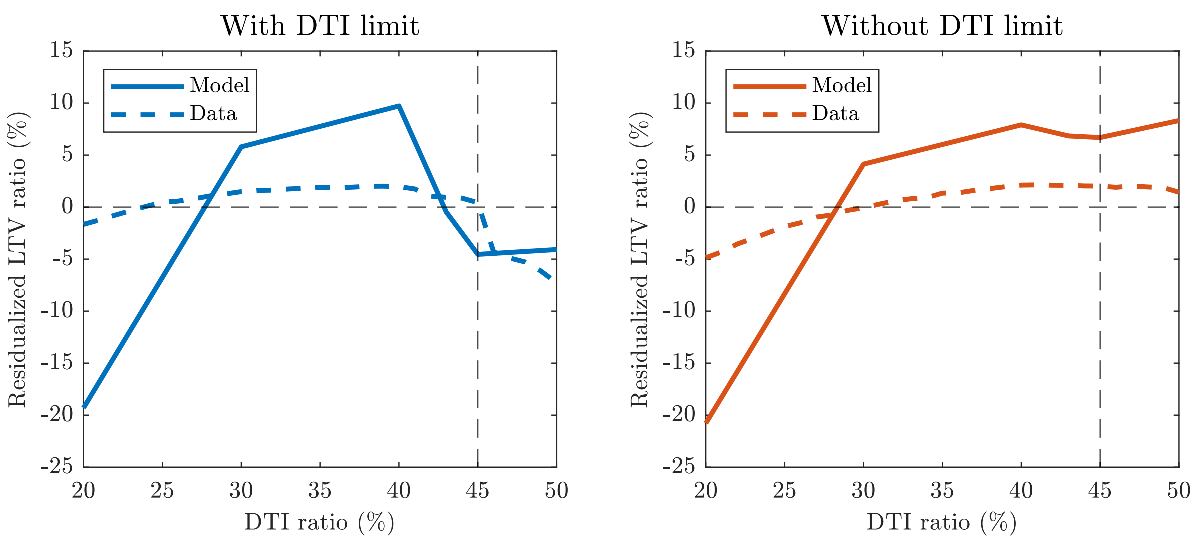
[42] See https://www.federalreserve.gov/econresdata/scf/files/scf2001list.sampleredesign9.pdf.

Figure 17. Default probabilities and DTI ratios in the model vs. data



Notes: "With DTI limit" refers to the baseline calibration of the model. "Without DTI limit" refers to an alternative calibration in which the default risk threshold for a high-DTI loan is fully relaxed. Default probability in the model is the *ex ante* probability that an age-j household in state ω optimally defaults at age $j+1$. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.

Figure 18. LTV ratios and DTI ratios in the model vs. data



Notes: "With DTI limit" refers to the baseline calibration of the model. "Without DTI limit" refers to an alternative calibration in which the default risk threshold for a high-DTI loan is fully relaxed. "LTV ratio" refers to LTV ratio at origination. I residualize model variables by demeaning because there is no aggregate uncertainty in the model. Data source: Freddie Mac Single Family Loan-Level Dataset.

what is found in the data.[43]

In Figure 18, I compare LTV ratios at origination in the model and the data for the pre- and post-policy periods. Consistent with the data, the model predicts that LTV ratios of mortgages above the current DTI limit are smaller than those of mortgages just on limit. In the absence of this constraint, LTV ratios remain elevated past the 45% cutoff. There are more mortgages with relatively small LTV ratios at origination in the model compared to the data. This likely reflects the fact that mortgages are the only form of debt available to households in the model. In reality, many households have access to unsecured credit to insure against smaller negative shocks to cash on hand. As a result, the model over-predicts the share of households who obtain mortgages with small balances at origination.[44] This quantitative feature of the model could be addressed by relaxing the no-borrowing constraint on the liquid asset to allow households some ability to smooth consumption through non-mortgage debt, then calibrating the borrowing limit on liquid wealth to match the observed share of households who carry positive unsecured debt.

B.6 Calibrating the cost of a non-qualified mortgage under Dodd-Frank

The Dodd-Frank Act of 2010 delegated the implementation of the ability-to-repay rule to the newly created Consumer Financial Production Bureau (CFPB). In January 2013, the Consumer Financial Protection Bureau (2013) announced the final rule and officially entered it into the *Federal Register*. I follow a cost-benefit analysis from the final rule to calibrate the value of γ_H, the foreclosure cost on a non-qualified mortgage to lenders.[45]

If a borrower who defaults on a non-QM brings a successful legal claim under the ability-to-repay rule, the lender who originated that mortgage is liable for up to three years of fees and finance charges; the borrower's legal expenses; and statutory damages under the Truth in Lending Act (TILA).[46] To compute the fees and finance charges owed by the lender, I calculate the mean of three years of interest payments on mortgages originated to borrowers with an above-average probability of default in the stationary distribution of the model. This amounts to 0.43 units of the numeraire consumption good. To this, I add the mortgage origination cost κ_m. The CFPB estimates that combined legal expenses of

[43]This result is robust to using other definitions of default—e.g., delinquent at any point in the life of the loan, more than 360 days delinquent, property has been repossessed by the lender—to construct the *ex post* probability of default in the data.

[44]The distribution of LTV ratios at origination in the model does exhibit a mass point at the statutory maximum of 85%, similar to the spikes at institutional LTV limits documented by Greenwald (2018).

[45]The final rule can be found at https://www.federalregister.gov/documents/2013/01/30/2013-00736/ability-to-repay-and-qualified-mortgage-standards-under-the-truth-in-lending-act-regulation-z.

[46]The Truth in Lending Act of 1968 requires that lenders disclose credit terms to consumers in a "meaningful way." Failure to do so can result in the lender being liable for statutory damages. See https://www.consumerfinance.gov/policy-compliance/rulemaking/regulations/1026/.

a lender and borrower are \$34,500 and additionally assumes that a borrower is rewarded \$4,000 in statutory damages under TILA. Together, the legal costs equal 74% of mean U.S. household income, equivalent to 1.03 units of the numeraire.[47] Finally, I add to this the housing transaction cost κ_h to account for costs generated by the foreclosure itself. In total, the resource loss suffered by a lender due to default on a non-qualified mortgage is 2.434.

In reality, not every borrower who is unable to repay their mortgage will bring a case against the responsible lender. That decision depends on, among other factors, whether the borrower lives in a judicial or non-judicial foreclosure state, as well as their willingness and/or ability to obtain legal representation. Nonetheless, substantial evidence suggests that mortgage lenders are worried about these regulations.[48] Fuster, Lo and Willen (2017) estimate that, between 2008 and 2014, the price of intermediation in the mortgage market increased by around 30 basis points per year and that this trend appears to be driven by increased net costs of mortgage servicing and heightened aversion to liability risk among lenders. Kim et al. (2018) document legal actions the GSEs and the U.S. federal government took after the Great Recession in response to improper loan originations, which could further deter lenders from originating mortgages with fewer legal protections.

B.7 Defining consumption-equivalent welfare change

Following Gete and Zecchetto (2018), I define composite consumption of an age-j household as

$$C_j \equiv \left(\alpha_j c_j^{1-\vartheta} + (1-\alpha_j)\, s_j^{1-\vartheta}\right)^{\frac{1}{1-\vartheta}}.$$

Consumption-equivalent welfare change $\Delta C_j(\omega)$ is the percent change in the composite consumption of age-j household in state ω needed to make them exactly indifferent between the stationary economies under the current and new policies. The assumption of CES preferences yields the closed-form solution

$$\Delta C_j(\omega) = \left[\left(\frac{\tilde{V}_j(\omega)}{V_j(\omega)}\right)^{\frac{1}{1-\sigma}} - 1\right] \times 100,$$

[47] Mean household labor income in the sub-sample of households in the 2016 SCF to which I calibrate the model is \$52,108 in 2013 CPI-U-RS adjusted dollars. Mean household labor income in the model is 2.23 units of the numeraire.

[48] The CFPB, for one, concedes that its estimate of litigation costs relies on "very conservative (likely unrealistic) assumptions." There is at least one known instance of a household bringing a successful suit under the ability-to-repay rule, *G. Elliot v. First Federal Community Bank*: https://law.justia.com/cases/federal/appellate-courts/ca6/19-3690/19-3690-2020-07-08.html.

where $V_j(\omega)$ and $\tilde{V}_j(\omega)$ are the value functions of an age-j household in state ω under the current and new policies, respectively. The aggregate welfare change in consumption-equivalent terms is therefore

$$\Delta C = \sum_{j=1}^{T} \left[\int \Delta C_j(\omega) \Lambda_j(d\omega) \right],$$

where $\Lambda_j(\omega)$ is the stationary distribution of age-j households over states under the initial policy.

C Numerical solution of the model

C.1 Simplifying the state space

Following Berger and Vavra (2015) and Kaplan and Violante (2014), I reduce the state space of the model by using the fact that, conditional on the household adjusting their house size, it is sufficient to track their home equity. In other words, the household only cares about the net cash on hand from selling their existing house and repaying outstanding mortgage debt, not its composition. I define the home equity e of a household with house size h, house price p, depreciation shock δ, mortgage debt m, and interest rate r_m as

$$e(h, \delta, p, m, r_m) \equiv (1-\delta) ph - (1+r_m) m. \tag{19}$$

This formulation significantly reduces the dimensionality of the state vector of a household who chooses to rent or chooses to obtain a new mortgage. Because the optimization problem associated with the latter is the most computationally intensive part of solving the household's problem, this helps to reduces overall computation time. Let $\omega_A \equiv (a, h, e, p, z)$ be the current state of an age-j household solving either of those optimization problems.[49]

A household who continues with an existing loan does not adjust their house size, so I continue to track their outstanding debt and mortgage interest rate independently. Their optimization problem remains in terms of the original vector of state variables, though, for notational consistency, I relabel it as $\omega_N \equiv (a, h, \delta, p, m, r_m, z)$.

Finally, note that, conditional on defaulting, home equity is irrelevant to the household's problem because default sets both their housing stock and outstanding debt to zero. Thus, I define the state vector of an age-j household who chooses to default as $\omega_D \equiv (a, p, z)$.

[49] Tracking h as a separate state variable is needed to determine if a household pays the housing adjustment cost. Tracking the depreciation shock δ is not, both because it is subsumed in the definition of home equity in Equation (19) and i.i.d. by assumption.

C.2 Redefining optimization problems

I rewrite the household's optimization problem for the simplified state space. The expected discounted lifetime utility of an age-j household who rents in state ω_A is

$$W_j^R(\omega_A) = \max_{x_j(\omega_A)} \left\{ V_j^R(\omega_A), V_j^M(\omega_A) \right\}. \tag{20}$$

The expected discounted lifetime utility of an age-j household who owns in state ω_N is

$$W_j^O(\omega_N) = \max_{x_j(\omega_N)} \left\{ V_j^R(\omega_A), V_j^M(\omega_A), V_j^P(\omega_N), V_j^D(\omega_D) \right\}, \tag{21}$$

where $\omega_A = (a, h, e(h, \delta, p, m, r_m), p, z)$.

If renting, a household solves

$$\begin{aligned} V_j^R(\omega_A) &= \max_{c,s,a'} \; u_j(c,s) + \beta \mathbb{E}_{p',z'|p,z} W_{j+1}^R(\omega_A') \\ &\quad \text{s.t.} \\ c + Rps + a' &\geq (1-\tau) y_j(z) + (1+r) a + e - \mathbb{1}_{h \neq 0} \kappa_h \\ a' &\geq 0 \\ \omega_A' &= (a', 0, 0, p', z'). \end{aligned} \tag{22}$$

Note that, if a household begins the next period as a renter, taking expectations over the depreciation shock is unnecessary because they will have a house size of zero.

If obtaining a new loan, a household solves

$$\begin{aligned} & V_j^M(\omega_A) = \max_{c,a',h',m'} \; u_j(c,h') + \beta \mathbb{E}_{\delta',p',z'|p,z} W_{j+1}^O(\omega_N') \\ & \quad \text{s.t.} \\ & c + a' + \mathbb{1}_{h' \neq h}(ph' - \kappa_h) \leq (1-\tau) y_j(z) + (1+r) a + e - (1 - \mathbb{1}_{h' \neq h}) ph + m' - \mathbb{1}_{m'>0} \kappa_m \\ & m' \leq \theta p h' \\ & \pi_{min,j}\left(m', r'_{m,j}(\omega_A)\right) \leq \lambda_j(\omega_A) y_j(z) \\ & \lambda_j(\omega_A) = \begin{cases} \lambda - \varsigma_j(\omega_A) & \text{if } \psi_j(\omega_A) > \Psi \\ \infty & \text{if } \psi_j(\omega_A) \leq \Psi \end{cases} \\ & a' \geq 0 \\ & \omega_N' = \left(a', h', \delta', p', m', r'_{m,j}(\omega_A), z'\right), \end{aligned} \tag{23}$$

where the expected probability of default is computed as

$$\psi_j(\omega_A) = \mathbb{E}_{\delta',p',z'|p,z}\left\{\mathbb{1}_{x_{j+1}(\omega_D)=D|x_j(\omega_A)=M}\right\}. \tag{24}$$

This formulation of the flow budget constraint allows me to compute the housing maintenance costs and loan repayment required by a mortgage refinance using the lower-dimensional state vector ω_A, despite the household not adjusting their housing stock. A mortgage refinance occurs when $\mathbb{1}_{h'\neq h} = 0$ and $\mathbb{1}_{m'>0} = 1$. Using the definition of home equity in Equation (19), the household's cash on hand is

$$(1-\tau)\,y_j(z) + (1+r)\,a - \delta ph - (1+r_m)\,m + m' - \mathbb{1}_{m'>0}\kappa_m.$$

Therefore, knowledge of h, e, and p is sufficient for backing out housing maintenance costs δph and loan repayment $(1+r_m)\,m$.

If making a payment on an existing loan, a household solves

$$\begin{aligned}
&V_j^P(\omega_N) = \max_{c,a',m'}\ u_j(c,h) + \beta\mathbb{E}_{\delta',p',z'|p,z}W_{j+1}^R(\omega_N') \\
&\qquad\text{s.t.} \\
&c + \delta ph + a' \leq (1-\tau)\,y_j(z) + (1+r)\,a - (1+r_m)\,m + m' \\
&m' \leq (1+r_m)\,m - \pi_{min,j}(m,r_m) \\
&a' \geq 0 \\
&\omega_N' = (a',h,\delta',p',m',r_m,z').
\end{aligned} \tag{25}$$

If defaulting, a household solves

$$\begin{aligned}
&V_j^D(\omega_D) = \max_{c,s,a'}\ u_j(c,s) - \xi + \beta\mathbb{E}_{p',z'|p,z}\left[\varphi W_{j+1}^R(\omega_A') + (1-\varphi)\,V_{j+1}^R(\omega_A')\right] \\
&\qquad\text{s.t.} \\
&c + Rps + a' \leq (1-\tau)\,y_j(z) + (1+r)\,a \\
&a' \geq 0 \\
&\omega_A' = (a',0,0,p',z').
\end{aligned} \tag{26}$$

The financial intermediary's problem presented in Section 4.3 remains unchanged. For notational consistency, I rewrite the present value of an existing mortgage held by an age-j

household in state ω_N as

$$\Pi_j(\omega_N) = \begin{cases} (1+r_m)\,m & \text{if repay} \\ (1-\delta)\,ph-\gamma & \text{if default} \\ (1+r_m)\,m-m'_j(\omega_N)+\dfrac{1}{1+r+\phi}\mathbb{E}_{\delta',p',z'|p,z}\Pi_{j+1}(\omega'_N) & \text{otherwise,} \end{cases} \tag{27}$$

where $\omega'_N = \left(a'_j(\omega_N), h'_j(\omega_N), \delta', p', m'_j(\omega_N), r'_{m,j}(\omega_N), z'\right)$. I rewrite the zero-profit condition on a mortgage originated to an age-j household in state ω_A as

$$(1+g)\,m'_j(\omega_A) = \frac{1}{1+r+\phi}\mathbb{E}_{\delta',p',z'|p,z}\Pi_{j+1}(\omega'_N), \tag{28}$$

where $\omega'_N = \left(a'_j(\omega_A), h'_j(\omega_A), \delta', p', m'_j(\omega_A), r'_{m,j}(\omega_A), z'\right)$.

C.3 Discretization

I use the Rouwenhorst method described by Kopecky and Suen (2010) to discretize the first order Markov processes for shocks to income z and house prices p. This method yields grids for each shock and a unique matrix of associated transition probabilities. The grid for house size h is $\left\{0, \underline{h}, \ldots, \bar{h}\right\}$, where, by definition, $h=0$ for existing renters and $h \in \left\{\underline{h}, \ldots, \bar{h}\right\}$ for existing owners. The points on the housing grid are set according to the procedure described in Section 5.2. The bounds on the grid for mortgage debt m are $\left[0, \theta\bar{p}\bar{h}\right]$, where $\bar{p}$ is the maximum possible house price. The bounds on the grid for the mortgage interest rate r_m are $[(1+g)(1+r+\phi)-1, \bar{r}_m]$. I confirm *ex post* that, in the stationary equilibrium of the model, the upper bound $\bar{r}_m$ does not bind. The grid for the depreciation shock δ is $\left\{\underline{\delta}, \bar{\delta}\right\}$. The grid for liquid assets a features more points clustered near the borrowing constraint. The bounds on the liquid asset grid are $[0, \bar{a}]$. I set $\bar{a}$ to $(1-\underline{\delta})\bar{p}\bar{h} + a_{mult}\bar{y}_j(z)$, where $a_{mult} > 0$ is a scalar and $\bar{y}_j(z)$ is the largest possible income realization in the model. I confirm *ex post* that the upper bound on liquid assets is not binding in the stationary distribution of the model. The bounds on the grid for home equity e are $[\underline{e}, \bar{e}]$, where $\underline{e} = \left(1-\bar{\delta}\right)\underline{p}\underline{h} - (1+\bar{r}_m)\,\bar{m}$ and $\bar{e} = (1-\underline{\delta})\,\bar{p}\bar{h} - (1+\underline{r}_m)\,\underline{m}$. The home equity grid contains a point at 0.

Choices for the three endogenous state variables a, m, and r_m—as well as values of e where applicable—are permitted to lie off the grid. I use linear interpolation to evaluate the value and policy functions at off-grid points when solving the model. I constrain the choice of h' to the house size grid in order to capture the indivisibility of housing as an asset. To construct the transition function for households over states $Q_j(\omega_N, \Omega_N)$ and compute the stationary distribution $\Lambda_j^*(\Omega_N)$, I interpolate the household's policy and value functions over finer grids for the three continuous endogenous state variables.

To evaluate expectations inside of the Bellman equations, I pre-compute integrals using the technique described by Judd et al. (2017). After I finish solving an age-j household's problem for all ω_A, ω_D, and ω_N, I define and compute the following for $j < T$:

$$\begin{aligned}
EW_j^R(\omega_A) &\equiv \mathbb{E}_{p',z'|p,z} \max\left\{V_j^R(a,h,e,p',z'), V_j^M(a,h,e,p',z')\right\} \\
EW_j^O(\omega_N) &\equiv \mathbb{E}_{\delta',p',z'|p,z} \max\left\{V_j^R(a,h,\tilde{e}',p',z'), V_j^M(a,h,\tilde{e}',p',z'),\right. \\
&\qquad \left. V_j^P(a,h,\delta',p',m,r_m,z'), V_j^D(a,p',z')\right\},
\end{aligned}$$

where $\tilde{e}' \equiv (1-\delta')p'h - (1+r_m)m$. $EW_j^R(\omega_A)$ and $EW_j^O(\omega_N)$ represent the household's continuation values conditional on holding the endogenous state variables fixed.

I therefore compute the value functions of an age-j household as

$$\begin{aligned}
V_j^R(\omega_A) &= u\left(c_j^R(\omega_A), s_j^R(\omega_A)\right) + \beta EW_{j+1}^R\left(a_j'^R(\omega_A), 0, 0, p, z\right) \\
V_j^M(\omega_A) &= u\left(c_j^M(\omega_A), h_j'^M(\omega_A)\right) \\
&\quad + \beta EW_{j+1}^O\left(a_j'^M(\omega_A), h_j'^M(\omega_A), \delta, p, m_j'^M(\omega_A), r_{m,j}'^M(\omega_A), z\right) \\
V_j^P(\omega_N) &= u\left(c_j^P(\omega_N), h\right) + \beta EW_{j+1}^O\left(a_j'^P(\omega_N), h, \delta, p, m_j'^P(\omega_N), r_m, z\right) \\
V_j^D(\omega_D) &= u\left(c_j^D(\omega_D), s_j^D(\omega_D)\right) + \beta\left[\varphi EW_{j+1}^R\left(a_j'^D(\omega_D), 0, 0, p, z\right)\right. \\
&\quad \left. + (1-\varphi) V_{j+1}^R\left(a_j'^D(\omega_D), 0, 0, p, z\right)\right],
\end{aligned}$$

where I obtain continuation values by interpolating $EW_{j+1}^R(\omega_A)$ and $EW_{j+1}^O(\omega_N)$ at the endogenous state variables' values in the next period. I also pre-compute integrals for the financial intermediary's problem, defining

$$E\Pi_j(\omega_N) \equiv \mathbb{E}_{\delta',p',z'|p,z}\Pi_j(a,h,\delta',p',m,r_m,z'),$$

such that the zero-profit condition is computed as

$$(1+g)\, m_j'^M(\omega_A) = \frac{1}{1+r+\phi} E\Pi_{j+1}\left(a_j'^M(\omega_A), h_j'^M(\omega_A), \delta, p, m_j'^M(\omega_A), r_{m,j}'^M(\omega_A), z\right).$$

C.4 Solution algorithm

Outer loop

1. For a given parameterization of the model and a measure of age-1 households, fix an initial guess for the proportional tax rate τ_0.

2. Solve for the recursive stationary equilibrium induced by τ_0 (see "Inner loop" section).

3. Given the equilibrium from the previous step, check if the government's budget constraint for credit guarantees in Equation (12) holds with equality. If not, update τ_0.

4. Repeat steps 2–3 until the government's budget balance is numerically close to 0.

Inner loop

1. Solve the problem of a household in the last period of life to obtain $V_T^R(\omega_A)$, $V_T^M(\omega_A)$, $V_T^P(\omega_N)$, and $V_T^D(\omega_D)$, along with all associated policy functions. By assumption, $m_T'(\omega_N) = 0$ and $m_T'(\omega_A) = 0$. Next, compute the present value of cash flows associated with a mortgage held by an age-T household in state ω_N. If the household repays their loan, then

$$\Pi_T(\omega_N) = (1 + r_m)\, m.$$

If the household defaults, then

$$\Pi_T(\omega_N) = (1 - \delta)\, ph - \gamma.$$

2. Use backward induction to solve for value functions in Equations (22), (23), (25), and (26) for ages $j < T$.

   (a) *Solving the problem of a household who chooses to rent, $V_j^R(\omega_A)$:*

      (i) This option is available to all households.

      (ii) The assumption of CES preferences over nondurable consumption and housing services implies

$$s = \left(\frac{1-\alpha}{\alpha R}\right)^{\frac{1}{\vartheta}} c. \tag{29}$$

      Use this expression to substitute out s in the household's problem, and use the budget constraint to substitute out c from the flow utility function.

      (iii) Solve for $V_j^R(\omega_A)$ and $a_j'^R(\omega_A)$ using Brent's method. Let $\underline{c}$ be the lowest possible value of nondurable consumption.[50] The requirement that $c \geq \underline{c}$ characterizes the set of feasible solutions,

$$\underline{a}' \leq a' \leq (1-\tau)\, y_j(z) + (1+r)\, a + e - \mathbb{1}_{h \neq 0} \kappa_h - \underline{c}\left[1 + \left(\frac{1-\alpha}{\alpha R^{1-\vartheta}}\right)^{\frac{1}{\vartheta}}\right]^{-1},$$

[50] In the computation, $\underline{c}$ is set to 0.001.

where the no-borrowing constraint on the liquid asset implies $\underline{a}' = 0$.

(iv) Find $c_j^R(\omega_A)$ from the flow budget constraint and use Equation (29) to obtain $s_j^R(\omega_A)$.

(v) By definition, $h_j'^R(\omega_A) = 0$ and $m_j'^R(\omega_A) = 0$. Because the household is not a debtor, $r_{m,j}'^R(\omega_A)$ can be set to any arbitrary value.

(b) *Solving the problem of a household who owns and obtains a new loan, $V_j^M(\omega_A)$:*

(i) This option is available to all households. When solving this problem for age-T households, $m_j'^M(\omega_A)$ is constrained to be 0 and the financial intermediary's problem is skipped. These households can still choose to adjust their housing stock, however.

(ii) Hold $r'_{m,j}(\omega_A)$ fixed. Loop through all feasible h'.[51] For each feasible h', use Nelder-Mead to solve for $a_j'^M(\omega_A)$, $m_j'^M(\omega_A)$, and $V_j^M(\omega_A)$. Maximum feasible debt is determined by the LTV and DTI limits,

$$\bar{m}' = \min\{\bar{m}'_{ltv}, \bar{m}'_{dti}\},$$

where

$$\bar{m}'_{ltv} \equiv \theta p h'$$

and

$$\bar{m}'_{dti} \equiv (\lambda - \varsigma_j(\omega_A))(1-\tau)y_j(z)\left[r'_{m,j}(\omega_A)\frac{\left(1+r'_{m,j}(\omega_A)\right)^{T-j}}{\left(1+r'_{m,j}(\omega_A)\right)^{T-j}-1}\right]^{-1}.$$

If the DTI limit is fully relaxed, then

$$\bar{m}' = \bar{m}'_{ltv}.$$

For a given $\bar{m}'$, the set of feasible solutions is characterized by

$$\begin{aligned}\underline{a}' \leq a' \leq\ & (1-\tau)y_j(z) + (1+r)a + e - (1-\mathbb{1}_{h'\neq h})ph + \bar{m}' - \kappa_m \\ & - \mathbb{1}_{h'\neq h}(ph' + \kappa_h) - \underline{c}\end{aligned}$$

[51]In this context, feasibility means that, for a given idiosyncratic state and $r'_{m,j}(\omega_A)$, h' is in the household's budget set and $c \geq \underline{c}$ assuming $m'_j(\omega_A) = \bar{m}'$, $a'_j(\omega_A) = \underline{a}'$, and $r'_{m,j}(\omega_A) = \underline{r}_m$—i.e., the household borrows the feasible maximum at the minimum interest rate while saving as little as possible.

and

$$\max\left\{\underline{c} + \mathbb{1}_{h'\neq h}\left(ph' + \kappa_h\right) + \underline{a}' + \kappa_m + \left(1 - \mathbb{1}_{h'\neq h}\right)ph - (1-\tau)\,y_j(z) \right.$$
$$\left. - (1+r)\,a - e, 0\right\} \leq m' \leq \bar{m}'.$$

Conditional on the DTI limit being fully relaxed, compute the expected probability of default $\psi_j(\omega_A)$ using Equation (24). Use the budget constraint to find $c_j^M(\omega_A)$. At the end of this step, select the value of h' (and policy functions implied that by that choice) that yields the highest expected lifetime utility for the household.

(iii) Given a solution to the household's problem found in the previous step, compute the financial intermediary's profit using Equation (28).

(iv) A bisection algorithm is used to find the break-even interest rate on a newly originated mortgage $r'_{m,j}(\omega_A)$. This algorithm exploits the fact that the financial intermediary's profit is increasing in $r'_{m,j}(\omega_A)$, all else equal, and searches over the interval $[\underline{r}_m, \bar{r}_m]$.

(A) The interest rate received by a household of age $T-1$ who obtains a new loan $r'_{m,T-1}(\omega_A)$ is $(1+g)(1+r+\phi)-1$. This follows from the fact that, if an age-T household repays their outstanding mortgage debt, then the zero profit condition is

$$\frac{1}{1+r+\phi}\left(1 + r'_{m,T-1}(\omega_A)\right) m'_{T-1} = (1+g)\, m'_{T-1},$$

implying $r'_{m,T-1}(\omega_A) = (1+g)(1+r+\phi) - 1$. Note that, if an age-$T$ household defaults, then, in equilibrium, an intermediary will not sell that household a mortgage contract in the previous period.

(B) If the financial intermediary's profit is negative when $r'_{m,j}(\omega_A) = \bar{r}_m$, then the option to obtain a mortgage is not available to the household in equilibrium and the expected probability of default $\psi_j(\omega)$ is set to 1. If the intermediary's profit is positive when $r'_{m,j}(\omega_A) = \underline{r}_m$, then the household borrows at the rate $(1+g)(1+r+\phi)-1$ and the expected probability of default $\psi_j(\omega)$ is set to 0. If the financial intermediary's profit is positive when $r'_{m,j}(\omega_A) = \bar{r}_m$ and negative when $r'_{m,j}(\omega_A) = \underline{r}_m$, then an interior solution exists and bisection is used to solve for the equilibrium interest rate that earns the lender zero profits on the loan in expectation.

(v) Execute steps (ii)–(iv) assuming the DTI limit does and does not apply in the household's problem. For each case, compute the household's expected probability of default using Equation (24). If the probability of default exceeds the default risk threshold conditional on the DTI requirement being relaxed, then the household's solution to this problem necessarily satisfies the statutory limit. Otherwise, select the case—i.e., with or without the DTI limit—that yields the highest value for the household.

(c) *Solving the problem of an owner who continues with an existing loan,* $V_j^P(\omega_N)$:

(i) This option is only available to existing homeowners ($h > 0$). Note that this problem is also solved by homeowners who do not have any debt. In this case, $m_j'^P(\omega_N) = 0$ and the owner only needs to solve for $a_j'^P(\omega_N)$.

(ii) Use the budget constraint to substitute c out of the flow utility function.

(iii) Solve for $a_j'^P(\omega_N)$, $m_j'^P(\omega_N)$, and $V_j^P(\omega_N)$ using Nelder-Mead. From the law of motion for mortgage debt, we have

$$\bar{m}' = (1 + r_m)\, m - \pi_{min,j}(m, r_m)\,,$$

where $\pi_{min,j}(m, r_m)$ is the minimum mortgage payment defined in Equation (2). The set of feasible solutions is characterized by

$$\underline{a}' \leq a' \leq (1 - \tau)\, y_j(z) + (1 + r)\, a - (1 + r_m)\, m + \bar{m}' - \delta p h - \underline{c}$$

and

$$\max\left\{\underline{c} + \underline{a}' + \delta p h - (1 - \tau)\, y_j(z) - (1 + r)\, a, 0\right\} \leq m' \leq \bar{m}'.$$

(iv) Use the budget constraint to find $c_j^P(\omega_N)$. By definition, $h_j'^P(\omega_N) = h$ and $r_{m,j}'^P(\omega_N) = r_m$.

(d) *Solving the problem of a borrower who defaults,* $V_j^D(\omega_D)$:

(i) This option is only available to existing borrowers ($h > 0$ and $m > 0$).

(ii) Because a household who defaults must rent in the current period, use Equation (29) to substitute out s from the flow utility function and the budget constraint.

(iii) Solve for $V_j^D(\omega_D)$ and $a_j'^R(\omega_D)$ using Brent's method. The set of feasible solutions is characterized by

$$\underline{a}' \leq a' \leq (1-\tau)\, y_j(z) + (1+r)\, a - \underline{c}\left[1 + \left(\frac{1-\alpha}{\alpha R^{1-\vartheta}}\right)^{\frac{1}{\vartheta}}\right]^{-1}.$$

(iv) Back $c_j^D(\omega_D)$ out from the flow budget constraint and use Equation (29) to obtain $s_j^D(\omega_D)$.

(v) By definition, $h_j'^D(\omega_D) = 0$ and $m_j'^D(\omega_D) = 0$. Because the household does not have any mortgage debt, $r_{m,j}'^D(\omega_D)$ can be set to any arbitrary value.

(e) Determine $W_j^R(\omega_A)$ and $W_j^O(\omega_N)$ using Equations (20) and (21).

(f) Compute $\Pi_j(\omega_N)$ using Equation (27).

(g) Compute $EW_j^R(\omega_A)$, $EW_j^O(\omega_N)$, and $E\Pi_j(\omega_N)$.

3. After solving for the stationary recursive equilibrium defined in Appendix B.1, interpolate value functions $\left\{V_j^R(\omega_N), V_j^M(\omega_N), V_j^P(\omega_N), V_j^D(\omega_N)\right\}$ and policy functions for endogenous state variables $\left\{a_j'^x(\omega_N), h_j'^x(\omega_N), m_j'^x(\omega_N), r_{m,j}'^x(\omega_N)\right\}$ for all $x \in \{R, M, P, D\}$ over finer grids for liquid assets a, mortgage debt m, and the interest rate r_m.[52] Determine housing tenure and loan adjustment choices using the interpolated valued functions. Compute policy functions for the control variables accordingly.

4. Given the finer value functions and policy functions for endogenous state variables in the previous step, as well as the probability distributions of δ, p, and z, construct the $(ns_{fine}/T) \times (ns_{fine}/T)$ transition matrix for the distribution of households over states $Q_j(\omega_N, \Omega_N)$ according to to Equation (17), where (ns_{fine}/T) denotes the number of fine grid points for a given j.

5. For a given $\mu_1(\omega_N)$ and $Q_j(\omega_N, \Omega_N)$, use the law of motion in Equation (16) to compute $\Lambda_{j+1}^*(\Omega_N)$ for all $j \in \{1, 2, \ldots, T-1\}$.

[52] In order to preserve the non-convexities introduced by the discrete choices in this model, I interpolate $V_j^M(\omega_N)$ and $\psi_j(\omega_N)$ twice, first assuming the DTI limit applies and again assuming it does not, then apply the underwriting technology to determine if the no-DTI-limit option is feasible. Note that, in this step, I define finer policy and value functions over the originally defined state vector ω_N.